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Bijections and symmetries for factorizations of the long cycle

We study the factorizations of the permutation $(1, 2, \dots, n)$ into k factors of given cycle types. Using the group algebra of the symmetric group, Jackson obtained for each k an elegant formula for counting these factorizations according to the number of cycles of each factor. In the cases $k = 2, 3$ Schaeffer and Vassilieva gave a combinatorial proof of Jackson's formula, and Morales and Vassilieva obtained more refined formulas exhibiting a surprising symmetry property. These counting results are indicative of a rich combinatorial theory which has remained elusive to this point, and it is the goal of this project to establish a series of bijections which unveil some of the combinatorial properties of these factorizations into k factors for all k . The first bijection is an instance of a correspondence of Bernardi between such factorizations and tree-rooted maps; certain graphs embedded on surfaces with a distinguished spanning tree. This is joint work with Olivier Bernardi.