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The Narrow Escape Problem for the Unit Sphere: Homogenization Limit, Optimal Trap Arrangements, and the N^2 Conjecture

A narrow escape problem is considered to calculate the mean first passage time (MFPT) needed for a Brownian particle to leave a domain through one of its N small boundary windows (traps). Narrow escape problems arise in chemical and cell-biological modeling. The MFPT satisfies a strongly heterogeneous Dirichlet-Neumann boundary value problem for the Poisson equation.

For the spherical domain, a procedure is established to calculate optimal arrangements of $N \gg 1$ equal small boundary traps that minimize the asymptotic MFPT. Based on observed characteristics of such arrangements, a remarkable property is discovered, that the sum of squared pairwise distances between optimally arranged N traps on a unit sphere is integer, equal to N^2. It is observed numerically for $2 \leq N \leq 1004$ with high precision. It is conjectured that this is the case for such optimal arrangements for all N. The conjecture is supported by an asymptotic estimate.

A dilute trap limit of homogenization theory when $N \to \infty$ is used to replace the strongly heterogeneous boundary value problem with a spherically symmetric Robin problem. For the latter, the exact solution is readily found. Parameters of the Robin homogenization problem are computed that capture the first four terms of the asymptotic MFPT. Close agreement of asymptotic and homogenization MFPT values is demonstrated. The homogenization approach provides a radically faster way to estimate the MFPT, since it is given by a simple formula, and does not involve computationally expensive global optimization to determine actual trap locations.

This is a joint work with D. Zawada.