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A unified perspective for Darmon points

Let $E_{/F}$ be an elliptic curve defined over a number field F, and let K/F be a quadratic extension. Assume that the sign of the functional equation of $L(E/K, s)$ is -1, so that the BSD conjecture predicts the existence of non-torsion points on $E(K)$.

If K/F is CM, Heegner points provide an abundant supply of points on E defined over abelian extensions of K. At the end of the last century Henri Darmon proposed a non-archimedean construction of local points on E in the case $F = \mathbb{Q}$ and K real quadratic, under a Heegner condition. Darmon also introduced an archimedean construction when F is totally real and K/F is "almost totally real". The non-archimedean construction was generalized by Matthew Greenberg to allow F to be a totally real field, while at the same time relaxing the Heegner condition; the archimedean setting was generalized in a similar way by Jerome Gartner. Finally, Mak Trifkovic considered a similar construction in the case of F being imaginary quadratic. All of these constructions predict the algebraicity and the field of definition of the resulting points, although almost nothing has been proven about them.

In a joint project with Xavier Guitart and Mehmet H. Sengun, we propose both archimedean and non-archimedean constructions of local points on E in the case of F having arbitrary signature, which recover all the above as particular cases. We will explain this unified construction in the context of the previous work, and provide evidence for the mixed-signature cases.