Connections Between Noncommutative Algebra and Geometry
Liens entre l'algèbre non commutative et la géométrie

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CHRIS BRAV, Institute for Advanced Study
Hamiltonian local models for symplectic derived stacks

We show that a derived stack with symplectic form of negative degree can be locally described in terms of generalised Darboux coordinates and a Hamiltonian cohomological vector field. As a consequence we see that the classical moduli stack of vector bundles on a Calabi-Yau threefold admits an atlas consisting of critical loci of regular functions on smooth varieties. If time permits, we discuss applications to the categorification of Donaldson-Thomas theory. This is joint work with subsets of Ben-Bassat, Bussi, Dupont, Joyce, and Szendroi.

RAGNAR BUCHWEITZ, University of Toronto Scarborough
Representation–infinite Algebras from Geometry

This is a report on joint work with Lutz Hille on the recent notion of higher preprojective algebras as introduced by Iyama and his collaborators. We show that a tilting object on a smooth projective variety X of dimension d has an endomorphism ring that is representation-infinite if, and only if, it pulls back to a tilting object on the affine canonical bundle over X if, and only if, that endomorphism algebra has minimal global dimension, equal to d, and no extensions in negative degrees against twists with negative powers of the canonical bundle.

The endomorphism ring of the pullback then yields the corresponding higher preprojective algebra. This proves, for example, that any foundation of a helix on a Fano variety gives rise to such a pair of a d-representation-infinite algebra and its accompanying higher $(d+1)$-preprojective algebra.

KENNETH CHAN, University of Washington
Noncommutative quadrics and $\mathbb{Z}^2$-graded algebras

In pursuit of new examples of Artin-Schelter (AS) regular algebras, Zhang-Zhang classified certain $\mathbb{Z}^2$-graded algebras which are double Ore extensions of AS regular algebras of dimension 2 into 26 families. Following Artin-Tate-Van den Bergh, we compute the point schemes of these algebras and re-interpret the Zhang-Zhang classification using geometric data. We also show that the associated noncommutative projective schemes are noncommutative quadric surfaces in the sense of Van den Bergh. This is joint work with Daniel Chan and Paul Smith.

HAILONG DAO, University of Kansas, Lawrence
On noncommutative crepant resolution of non-Gorenstein singularities

Let R be a normal domain. Recall that a noncommutative crepant resolution (NCCR) of R is the endomorphism ring A of a reflexive R-module M such that A is Cohen-Macaulay over R with global dimension equal to the Krull dimension of R. In this talk we discuss a necessary and sufficient condition for existence of NCCRs when R is Cohen-Macaulay containing an algebraically closed field of characteristic 0. The result allows us to transfer the problem of finding NCCRs to the canonical cover of R. This is joint work with Osamu Iyama and Ryo Takahashi.

ELEONORE FABER, University of Toronto
Non-commutative resolutions of non-normal rings

In this talk we consider non-commutative analogs of resolutions of singularities for not-necessarily normal commutative rings. These non-commutative resolutions of commutative rings R are endomorphism rings of certain R-modules of finite global dimension. We will in particular consider Van den Bergh's non-commutative crepant resolutions (NCCRs) and non-commutative resolutions (NCRs) as recently defined by Dao, Iyama,Takahashi and Vial. We give some conditions and obstructions for existence of NC(C)Rs over certain non-normal rings. This is joint work with H. Dao and C. Ingalls.

ELLEN KIRKMAN, Wake Forest University
Finiteness conditions on the Ext algebra of a monomial algebra

Let k be a field and let A be a monomial k-algebra, $A = T(V)/I$, where $T(V)$ is a finitely generated tensor k-algebra and I is a set of monomials in $T(V)$. We associate a finite graph $\Gamma(A)$ to A, and use $\Gamma(A)$ to characterize finiteness properties of $\mathrm{Ext}_A(k,k)$, the Yoneda Ext algebra of A, including finite Gelfand-Kirillov dimension, the noetherian property, and finite generation of $\mathrm{Ext}_A(k,k)$. (Joint work with Andrew Conner, James Kuzmanovich, and W. Frank Moore)

DANIEL KRASHEN, University of Georgia
Derived categories of torsors for Abelian varieties

For curves C_1, C_2 of genus not equal to 1 over arbitrary fields, it is known that the bounded derived categories of C_1 and C_2 are equivalent if and only if the curves are isomorphic. The case of genus 1 is much richer, and in this talk I'll describe some recent joint work with Ben Antieau and Matthew Ward on derived equivalence for genus 1 curves over arbitrary fields as well as generalizations to torsors for Abelian varieties.

TOM LENAGAN, University of Edinburgh
Totally nonnegative matrices

A real matrix is *totally nonnegative* if each of its minors is nonnegative, and is *totally positive* if each minor is greater than zero. We will outline connections between the theory of total nonnegativity and the prime spectrum of the algebra of quantum matrices, and will discuss some new and old results about total nonnegativity which may be obtained using methods derived from quantum matrix methods. Most of the material is joint work with Stéphane Launois and Ken Goodearl.

GRAHAM LEUSCHKE, Syracuse University
Pieri maps and the bound Young quiver

The irreducible polynomial representations $L^\alpha V$ of $\mathrm{GL}(V)$ are well-known to be indexed by partitions α with at most $\dim(V)$ parts. The Pieri rules for decomposing the tensor products $V \otimes L^\alpha V$ and $V^* \otimes L^\alpha V$ into irreducibles defines, up to some choices of scalars, a system of split inclusions between those representations related by adding or removing a single box from the partitions. The scalars cannot be chosen with complete freedom; in particular there are some unavoidable non-commutativity relations among the Pieri maps. We build a quiver out of the data of partitions, maps, and relations, and show that the path algebra of this bound quiver is a non-commutative desingularization of a generic determinantal ring.

BRENT PYM, McGill University
Quantum deformations of projective three-space

The classification of noncommutative versions of projective three-space (in the form of four-dimensional Artin–Schelter regular algebras) is an important open problem in noncommutative projective geometry. I will discuss some recent progress on this question, in the form of an explicit description of the possible Calabi–Yau deformations of the polynomial ring. The approach uses results of Dolgushev and Kontsevich on deformation quantization, together with some Poisson geometry, to reduce the problem to Cerveau and Lins Neto's classification of degree-two foliations of projective space.

DAVID SALTMAN, CCR-Princeton
Persistent Fields

In 1976 Murray Schacher and Burt Fein showed the following. Suppose D/F is a division algebra, L/F a Galois extension of fields, and every maximal subfield of D contains an isomorphic copy of L/F. Then $L = F(\sqrt{-1})$ and D contains $(-1,-1)_F$ the Hamilton quaternions. Louis Rowen and I revisited this sort of question responding to some questions of Andrei Rapinchuk concerning linear algebraic groups. I will explain this connection and give our strengthening of the Fein Schacher result. Because of symplectic and orthogonal groups, we also prove parallel results for division algebras with involution.

CHELSEA WALTON, Massachusetts Institute of Technology
PBW deformations of smash product algebras from Hopf actions on Koszul algebras

This talk concerns the actions of Hopf algebras H on Koszul algebras B. The aim of this work is to provide necessary and sufficient conditions for a certain filtered algebra to be a Poincaré-Birkhoff-Witt (PBW) deformation of the smash product algebra $B\#H$. Many ring-theoretic properties are preserved under PBW deformation, and the representation theory of examples of such deformations has been an active area of research (e.g. symplectic reflection algebras, rational Cherednik algebras). Our theorem encompasses known results on PBW deformations in the literature and we provide many interesting examples, both old and new, illustrating our result. This is joint work with Sarah Witherspoon.

MILEN YAKIMOV, Louisiana State University
Cluster algebra structures on quantum double Bruhat cells

Cluster algebras were defined by Fomin and Zelevinsky for the purposes of the axiomatic study of canonical bases and total positivity. An important open problem for these applications was the Berenstein-Zelevinsky conjecture that the quantized coordinate rings of all double Bruhat cells in complex simple Lie groups admit upper quantum cluster algebra structures. We will give a prove of this conjecture, which also shows that each of the upper quantum cluster algebras coincides with the corresponding quantum cluster algebra. This is a joint work with Ken Goodearl, UC Santa Barbara.