Number Theory
Théorie des nombres
(Org: **Damien Roy** (University of Ottawa))

SANDRO BETTIN, CRM
The reciprocity formula for the twisted second moment of Dirichlet L-functions

Conrey noticed that the second moment of Dirichlet L-functions satisfies an approximate reciprocity relation. We will investigate further this formula, showing how to extend it to an exact formula. Moreover we will highlight the connection between the twisted second moment and the Estermann function.

DANIEL FIORILLI, University of Michigan
A probabilistic study of the explicit formula

Probabilistic arguments as well as numerical data suggests that for large moduli, the error term in the prime number theorem for arithmetic progressions is much smaller than what GRH predicts. Based on such arguments Montgomery formulated a conjecture which fits the numerical data and which implies several well believed conjectures for primes in arithmetic progressions, such as the Elliot-Halberstam Conjecture and the equidistribution of primes up to x modulo q as soon as x exceeds $q^{1+\epsilon}$. In formulating Montgomery's Conjecture, one should assume that Dirichlet L-functions do not vanish at the central point. We will show how to reformulate the conjecture without this assumption, and show how the modified conjecture implies that almost all Dirichlet L-functions do not vanish at the central point. We will then show that these arguments can be modified to other families of L-functions, and will focus on families of elliptic curve L-functions. In work in progress, our results are that a conjecture analogous to Montgomery's implies that the average analytic rank of the curves in the family is bounded above by $1/2$, and in some cases we can show that exactly half of the curves have algebraic rank 0, and the remaining half have algebraic rank 1.

EYAL GOREN, McGill University
Modular forms on Picard modular surfaces

We shall discuss modular forms on Picard modular surfaces. This is joint work with Ehud de Shalit (Hebrew University).

KEVIN HARE, University of Waterloo
Sporadic Reinhardt polygons

Let n be a positive integer, not a power of two. A *Reinhardt polygon* is a convex n-gon that is optimal in three different geometric optimization problems: it has maximal perimeter relative to its diameter, maximal width relative to its diameter, and maximal width relative to its perimeter. There is a correspondence between Reinhardt polygons and polynomials with specific divisibility properties and with tight restrictions on their coefficients. This correspondence is useful in the construction of such polygons. For almost all n, there are many Reinhardt polygons with n sides, and many of them exhibit a particular periodic structure. While these periodic polygons are well understood, for certain values of n, additional Reinhardt polygons exist that do not possess this structured form. We call these polygons *sporadic*. We completely characterize the integers n for which sporadic Reinhardt polygons exist, showing that these polygons occur precisely when $n = pqr$ with p and q distinct odd primes and $r \geq 2$. We also prove that a positive proportion of the Reinhardt polygons with n sides are sporadic for almost all integers n, and we investigate the precise number of sporadic Reinhardt polygons that are produced for several values of n by a construction that we introduce.

KEVIN HENRIOT, Université de Montréal
Modelling in additive combinatorics

Consider a subset A of the first n integers. By work of Sanders and Croot, Laba, Sisask, we know that if A has density at least $(\log n)^{-1+\varepsilon}$, then A contains a three-term arithmetic progression, and $A+A$ contains a 'long' three-term arithmetic progression. In this talk, we consider generalizations of these two statements to the case where A is a finite subset of an arbitrary abelian group, subject only to a condition of small doubling. The method we use is a refinement of the original modelling technique of Ruzsa, using recent developments on the polynomial Freiman-Ruzsa conjecture.

JING-JING HUANG, University of Toronto
Metric Diophantine approximation on manifolds

In the area of metric Diophantine approximation, there has been undergoing some fascinating interplays between number theory, dynamic system and harmonic analysis. The subject was initiated by Khinchine and Jarnik who establish respectively the Lebesgue and Hausdorff theories for simultaneous Diophantine approximation on Euclidean spaces. A fundamental question was subsequently raised by Baker and Sprindzuk that whether points on a generic manifolds satisfy the same Diophantine property which asserts that almost all or almost no points on the manifolds can be approximated well by rational points according as whether a certain volume sum diverges or converges. Some tremendous progress have been made towards the full solution of this conjecture by Kleinbock-Margulis, Beresnevich, Vaughan-Velani etc in the last 15 years. In this talk, I will survey these progress and present some very recent development made by the author.

THOMAS HULSE, Queen's University
Some Results of Shifted Sums

Knowledge about shifted convolution sums can be used to bound different aspects of automorphic L-functions and thus make piecemeal progress toward the General Lindelöf Hypothesis. In joint work with Jeffrey Hoffstein, joint work with E. Mehmet Kıral, Chan Ieong Kuan and Li-Mei Lim, and my own thesis work, we investigated the application of a different sort of truncated Poincaré series, proposed by Hoffstein, in our study of particular shifted sums. These sums have in turn been used to obtain non-trivial asymptotics of triple sums of Fourier coefficients of classical holomorphic cusp forms, a Burgess-type bound for twisted automorphic L-functions, and a smoothed count of square discriminants with bounded coefficients.

PATRICK INGRAM, Colorado State University
The filled Julia set of a Drinfeld module

The theory of Drinfeld modules over global function fields has many interesting parallels with the theory of elliptic curves over number fields. On the other hand, over a local function field Drinfeld modules can be seen as a type of polynomial dynamical system. In this talk, we will use this dynamical perspective to introduce the filled Julia set of a Drinfeld module over the adele ring of a global function field, and its component module, and exhibit some striking similarities with the structure of the component module of an elliptic curve.

PAYMAN KASSAEI, McGill University
Companion forms in parallel weight one

A complication faced in the study of optimal weights in Serre's conjecture on the modularity of mod-p Galois representations is the existence of companion forms. Serre conjectured that a mod-p modular form has a companion form exactly when its Galois representation is tamely ramified at p. This was proved by Gross modulo some unchecked compatibilities, and later by Coleman-Voloch (almost) unconditionally.

This work has been generalized over totally real fields in many cases. In this talk, I will explain joint work with Toby Gee on the construction of companion Hilbert modular forms in parallel weight one. The proof entails a mix of modularity lifting results and geometry.

YOUNESS LAMZOURI, York University
A-points of the Riemann Zeta function

The complex roots, $s = \sigma + it$, to the equation $\zeta(s) = a$, where a is non-zero complex number, are known as a-points of the Riemann zeta function. In this talk, I will present joint work with Steve Lester and Maksym Radziwill in which we obtain the first effective error term for the number of a-points in a strip $1/2 < \sigma_1 < \sigma < \sigma_2 < 1$. Previously only an asymptotic estimate was available due to a result of Bohr and Jessen from 1932.

ANTONIO LEI, McGill University
Iwasawa Theory of Abelian Varieties at Supersingular Primes

In this talk, I will explain how we could generalize works of Kobayashi, Pollack and Sprung on supersingular elliptic curves to abelian varieties using p-adic Hodge theory. In particular, I will explain how to construct a logarithmic matrix that we could use to decompose Perrin-Riou's exponential map and how this leads to a natural definition of Kobayashi/Sprung-type Selmer groups. This is joint work with Kazim Buyukboduk.

MARC MASDEU, Columbia University
A unified perspective for Darmon points

Let $E_{/F}$ be an elliptic curve defined over a number field F, and let K/F be a quadratic extension. Assume that the sign of the functional equation of $L(E/K, s)$ is -1, so that the BSD conjecture predicts the existence of non-torsion points on $E(K)$.

If K/F is CM, Heegner points provide an abundant supply of points on E defined over abelian extensions of K. At the end of the last century Henri Darmon proposed a non-archimedean construction of local points on E in the case $F = \mathbb{Q}$ and K real quadratic, under a Heegner condition. Darmon also introduced an archimedean construction when F is totally real and K/F is "almost totally real". The non-archimedean construction was generalized by Matthew Greenberg to allow F to be a totally real field, while at the same time relaxing the Heegner condition; the archimedean setting was generalized in a similar way by Jerome Gartner. Finally, Mak Trifkovic considered a similar construction in the case of F being imaginary quadratic. All of these constructions predict the algebraicity and the field of definition of the resulting points, although almost nothing has been proven about them.

In a joint project with Xavier Guitart and Mehmet H. Sengun, we propose both archimedean and non-archimedean constructions of local points on E in the case of F having arbitrary signature, which recover all the above as particular cases. We will explain this unified construction in the context of the previous work, and provide evidence for the mixed-signature cases.

JAMES MAYNARD, CRM, Universite de Montreal
The number of prime factors of n(n+2)(n+6)

It is believed that there should be infinitely many integers n for which n, $n+2$ and $n+6$ are all primes. Unfortunately proving this seems to be well beyond our current capabilities.

We will show that there are infinitely many integers n for which n, $n + 2$ and $n + 6$ together have at most 7 prime factors in total. Our key new idea is a switching principle which allows us to combine sieves of different dimensions to get stronger estimates than were previously available.

DAVID MCKINNON, University of Waterloo
Rational points on cubic surfaces

It is a famous unsolved problem in Diophantine geometry to get a good estimate for the number of rational points of bounded height on a smooth cubic surface. In this talk, I will not solve this problem by discussing how rational and algebraic points on a cubic surface approximate one another.

CAMERON L. STEWART, University of Waterloo
On a refinement of the abc conjecture

We shall discuss joint work with Robert and Tenenbaum on a proposed refinement of the well known abc conjecture.

NAOMI TANABE, Queen's University
Notes on Fourier Coefficients of Hilbert Modular Forms

We report on some properties of Fourier coefficients for Hilbert modular forms. In particular, we discuss some new results on their sign changes. This is a joint work with Jaban Meher.

GARY WALSH, Tutte Institute and U. Ottawa
The arithmetical structure of terms in Lucas sequences

We present some results and discuss open problems on the subject of the title.