MING MEI, Champlain College St.-Lambert and McGill University

Stability of oscillating wavefronts for time-delayed reaction-diffusion equation

This talk is concerned with Nicholson's blowflies equation. It is known that, when the ratio of birth rate coefficient and death rate coefficient satisfies $1 < \frac{p}{d} \le e$, the equation is monotone, and possesses monotone traveling wavefronts, which are intensively studied in the previous research. However, when $\frac{p}{d} > e$, the equation losses its monotonicity, and its traveling waves are oscillatory when the time-delay r or the wave speed c is large, which causes the study of stability of these non-monotone traveling waves to be challenging. In this talk, we present that, when $e < \frac{p}{d} \le e^2$, all traveling waves $\phi(x+ct)$ with $c \ge c_* > 0$, including the critical waves and non-critical waves, are asymptotically stable, and the non-critical waves are exponentially stable, where $c_* > 0$ is the minimum wave speed. In this case, we allow all traveling waves to be any, monotone or non-monotone with any speed $c \ge c_*$, and any size of the time-delay $r > 0$; While, when $\frac{p}{d} > e^2$ with a small time-delay $r < \left[\pi - \arctan\sqrt{\ln\frac{p}{d}(\ln\frac{p}{d} - 2)}\ \right]/d\sqrt{\ln\frac{p}{d}(\ln\frac{p}{d} - 2)}$, then all traveling waves $\phi(x+ct)$ with $c \ge c_* > 0$ are stable, too, and the non-critical waves are exponentially stable. A new technique with a nonlinear Halanay's inequality is introduced to establish weighted L^2 energy estimates. As a corollary, we also prove the uniqueness of traveling waves in the case of $\frac{p}{d} > e^2$, which was open as we know. Some numerical simulations in different cases are also carried out, which either confirm and support our theoretical results, or open up some new phenomena for future research.