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Contractions of 2D 2nd order quantum superintegrable systems and the Askey scheme for hypergeometric orthogonal polynomials

A quantum superintegrable system is an integrable n-dimensional Hamiltonian system on a Riemannian manifold with potential: $H = \Delta_n + V$ that admits 2n-1 algebraically independent partial differential operators commuting with the Hamiltonian, the maximum number possible. A system is of order L if the maximum order of the symmetry operators, other than H, is is L. For $n = 2$, $L = 2$ all systems are known. There are about 50 types but they divide into 12 equivalence classes with representatives on flat space and the 2-sphere. The symmetry operators of each system close to generate a quadratic algebra, and the irreducible representations of this algebra determine the eigenvalues of H and their multiplicity. All the 2nd order superintegrable systems are limiting cases of a single system: the generic 3-parameter potential on the 2-sphere, $S9$ in our listing. Analogously all of the quadratic symmetry algebras of these systems are contractions of $S9$. The irreducible representations of $S9$ have a realization in terms of difference operators in 1 variable. It is exactly the structure algebra of the Wilson and Racah polynomials! By contracting these representations to obtain the representations of the quadratic symmetry algebras of the other less generic superintegrable systems we obtain the full Askey scheme of orthogonal hypergeometric polynomials. This relationship provides great insight into the structure of special function theory and directly ties the structure equations to physical phenomena.

Joint work with Ernie Kalnins and Sarah Post