ALEXANDER BIHLO, Centre de recherches mathématiques, Université de Montréal
Invariant discretization schemes

Geometric numerical integration is a recent field in the numerical analysis of differential equations. It aims at improving the quality of the numerical solution of a system of differential equations by preserving qualitative features of that system. Such qualitative feature can be conservation laws, a Hamiltonian or variational structure or a nontrivial point symmetry group. While quite some effort has been put in the construction of conservation law preserving and Hamiltonian discretization schemes, the problem of finding invariant numerical integrators is more recent and less investigated. The main obstacle one faces when constructing symmetry-preserving approximations for evolution equations is that these discretizations generally require the usage of moving meshes. Grids that undergo an evolution in the course of numerical integration pose several theoretical challenges, especially in the multi-dimensional case.

In this talk we will present three possible strategies to overcome the problem with invariant moving meshes and thus address the practicability of symmetry-preserving discretization schemes. These ways are the discretization in computational coordinates, the use of invariant interpolation schemes and the formulation of invariant meshless schemes. The different strategies will be illustrated by presenting the results obtained from invariant numerical schemes constructed for the linear heat equation, a diffusion equation and the system of shallow-water equations.