TATCHAI TITICHETRAKUN, UBC

Corners in dense subset of $\mathbb{P}^d$

Furstenberg-Katnelson's Theorem states that if A is a subset of $\mathbb{Z}^d$ with positive upper density then for any finite subset F of $\mathbb{Z}^d$, A contains an affine image of F. We wish to prove analogue theorem in prime tuples $\mathbb{P}^d$ where positive upper density is replaced by positive relative upper density in $\mathbb{P}^d$. This is partially done by Magyar and Cook in the case that no two points in F have the same orthogonal projection to any coordianate axis ; when we count such configurations in that case, $\mathbb{P}^d$ behaves like a random subset of $\mathbb{Z}^d$ but this is not true in general since $\mathbb{P}^d$ has direct product structure and the natural majorant of $\mathbb{P}^d$ cannot be pseudorandom. In this talk, we will discuss how to use hypergraph approach, Green-Tao measure and Gowers's Transference Principle to deal with the case that F is the corner i.e., simplex of the form

$$\{(x_1, .., x_d), (x_1 + s, x_2, .., x_d), ..., (x_1, ..., x_d + s)\}, s \neq 0.$$

We expect that the same method should also work for any finite set F. This is a joint work with Akos Magyar.