Mathematical Physics - Random Matrices and Integrable Systems
Physique mathématique - matrices aléatoires et systèmes intégrables
(Org: **Marco Bertola** and/et **Dmitry Korotkin** (Concordia))

MARCO BERTOLA, Concordia University
Universality in Unitary Random matrix models

It is very well known that for Hermitean matrices the scaling limit of the eigenvalue statistics is a determinantal point field with kernel given by the sine kernel in the bulk and the Airy kernel (generally) at the edge.

We consider the normal matrix model with external field $U = Tr(MM^\dagger) - Tr(Harm(M))$ where $Harm$ stands for a (locally) harmonic function. We give a conjectural form for the strong asymptotic of the corresponding orthogonal polynomials. This form has been verified in a few outstanding cases. We then show how to use this conjectural form to prove universality in the bulk and on the boundary of the support region for the asymptotic location of eigenvalues, where the limiting kernel are simple expressions in terms of exponentials and the complementary error function. Thus we prove universality in all cases where the conjecture has been, or will be, verified. The proof does not require anything more than some general features.

PAVEL BLEHER, Indiana University-Purdue University Indianapolis
Exact solution of the six-vertex model with DWBC. Critical line between disordered and antiferroelectric phases

We obtain the large N asymptotics of the partition function Z_N of the six-vertex model with domain wall boundary conditions on the critical line between the disordered and antiferroelectric phases. Using the weights $a = 1 - x, b = 1 + x, c = 2, |x| < 1$, we prove that, as $N \to \infty$, $Z_N = CF^{N^2} N^{1/12} \left(1 + O(N^{-1})\right)$, where F is given by an explicit expression in x and the x-dependency in C is determined. Our result gives a complete proof and substantially strengthens the one given in the physics literature by Bogoliubov, Kitaev and Zvonarev. Furthermore, we prove that the free energy exhibits an infinite order phase transition between the disordered and antiferroelectric phases. Our proofs are based on the large N asymptotics for the underlying orthogonal polynomials which involve a non-analytical weight function, the Deift-Zhou nonlinear steepest descent method to the corresponding Riemann-Hilbert problem, and the Toda equation for the tau-function. This is a joint work with Thomas Bothner.

ANTON DZHAMAY, University of Northern Colorado
Discrete Hamiltonian Structure of Schlesinger Transformations

Schlesinger transformations are algebraic transformations of a Fuchsian system that preserve its monodromy representation and act on the characteristic indices of the system by integral shifts. One of the main reasons for studying these transformations is the relationship between Schlesinger transformations and discrete Painlevé equations; this is also the main motivation behind our work. In this talk we show how to write an elementary Schlesinger transformation as a discrete Hamiltonian system w.r.t. the standard symplectic structure on the space of Fuchsian systems. We also show how such transformations reduce to discrete Painlevé equations by computing two explicit examples, d-$P\big(D_4^{(1)}\big)$ (or difference Painlevé V) and d-$P\big(A_2^{(1)*}\big)$. In considering these examples we also illustrate the role played by the geometric approach to Painlevé equations not only in determining the type of the equation, but also in studying the relationship between different explicit forms of equations of the same type.

This is a joint work with Tomoyuki Takenawa (Tokyo University of Marine Science and Technology) and Hidetaka Sakai (The University of Tokyo).

JOHN HARNAD, Concordia University and CRM
Finite Dimensional Tau Functions

We show how a microcosm of the τ-function approach to the KP hierarchy developed by the Kyoto group, consisting of solutions having a finite number of degrees of freedom, may be studied within the setting of finite dimensional Grassmannnians. This gives both a Grassmannian and fermionic interpretation of the determinantal formula of Gekhtman and Kasman, and makes evident the origin of the "rank-1" condition characterizing finite dimensional reductions. In particular, this includes the well-known cases of polynomial τ-functions, those associated to Calogero-Moser pole dynamics, multisolitions and their degenerations. It also sheds light on the recently introduced notion of "convolution flows". (Based on joint work with F. Balogh and T. Dinis da Fonseca)

DMITRY KOROTKIN, Concordia University
Baker-Akhiezer spinor and Bergman tau-function on moduli spaces of meromorphic differentials

We derive variational formulas of Rauch-Ahlfors type on moduli spaces of meromorphic differentials on Riemann surfaces. In particular, we show that the derivatives of the Szegö kernel with respect to homological coordinates on these spaces are expressed via Hirota derivative of that kernel. This formula is used to derive variational formulas for the Baker-Akhiezer kernel, which in particular encode the KP-type hierarchies, as well as dependence of the baker-Akhiezer kernel on the moduli of the Riemann surface. We also define Bergman tau-function on these spaces, compute it in several important special cases and describe it as a section of an appropriate line bundle; this allows to express the Hodge class on these moduli spaces in terms of the tautological class.

ANDREW MCINTIRE, Bennington College
Chern-simons invariant of infinite volume hyperbolic 3-manifolds

We define a Chern-Simons invariant for a certain class of infinite volume hyperbolic 3-manifolds. We then prove an expression relating the Bergman tau function on a cover of the Hurwitz space, to the lifting of the function F defined by Zograf on Teichmüller space, and another holomorphic function on the cover of the Hurwitz space which we introduce. If the point in cover of the Hurwitz space corresponds to a Riemann surface X, then this function is constructed from the renormalized volume and our Chern-Simons invariant for the bounding 3-manifold of X given by Schottky uniformization, together with a regularized Polyakov integral relating determinants of Laplacians on X in the hyperbolic and singular flat metrics. Combining this with a result of Kokotov and Korotkin, we obtain a similar expression for the isomonodromic tau function of Dubrovin. We also obtain a relation between the Chern-Simons invariant and the eta invariant of the bounding 3-manifold, with defect given by the phase of the Bergman tau function of X.

ANTHONY METCALFE, KTH, Royal Institute of Technology, Sweden
Universality classes of lozenge tilings of a polyhedron

A regular hexagon can be tiled with lozenges of three different orientations. Letting the hexagon have sides of length n, and the lozenges have sides of length 1, we can consider the asymptotic behaviour of a typical tiling as n increases. Typically, near the corners of the hexagon there are regions of "frozen" tiles, and there is a "disordered" region in the center which is approximately circular.

More generally one can consider lozenge tilings of polyhedra with more complex boundary conditions. The local asymptotic behaviour of tiles near the boundary of the equivalent "frozen" and "disordered" regions is of particular interest. In this talk, we shall discuss work in progress in which we classify necessary conditions under which such tiles behave asymptotically like a determinantal random point field with the Airy kernel, and also with the Pearcey kernel. We do this by considering an equivalent interlaced discrete particle system.

CHRISTOPHER SINCLAIR, University of Oregon
Kernel Asymptotics for the Mahler Ensemble of Real Polynomials

The Mahler measure of a polynomial is the absolute value of the lead coefficient times the product of the absolute values of the roots outside the unit circle. The set of degree N polynomials with Mahler measure at most 1 forms a bounded subset of $\mathbb{R}^{N+1}$. The roots of polynomials chosen uniformly from this region yields a Pfaffian point process on the complex plane similar to that of Ginibre's real ensemble but with a different (sub-exponential) weight. The limiting density of roots is uniform measure on the unit circle, and we discuss the scaling limits for the matrix kernel in a neighborhood of a point on the unit circle. New phenomena appear in a neighborhood of 1, since the spectrum consists of both real roots and complex conjugate pairs. Relationships with the related determinantal ensemble (of roots of complex polynomials) will be discussed as well as an electrostatic and matrix model for the ensemble.

JACEK SZMIGIELSKI, University of Saskatchewan
Two applications of Cauchy biorthogonal polynomials

Cauchy biorthogonal polynomials were introduced by M. Bertola, M. Gekhtman and the speaker. They appeared originally in the computations of peakon solutions to the Degasperis-Processi equation. In this talk I will review basic properties of this class of polynomials and describe the highlights of two recent applications; one, to the solution of certain inverse problem associated to a system of equations put forward by Geng and Xue, the other, to the computation of the correlation kernels for the Cauchy two-matrix model with Laguerre type one-body interactions.

BALINT VIRAG, University of Toronto
Random operators at the edge

The density of states of a random GUE matrix at the edge behaves like $x^{1/2}$. The large-n limit of this matrix is the Stochastic Airy Operator, whose ground state has Tracy-Widom distribution. The Painleve II formulas for this distribution can be derived using the random operator.

The density of states of some natural classes of matrix models have different power law at the edge. I will describe the conjectured limiting operators and state some open problems about their behavior.