Geometry of Algebraic Varieties
Géométrie des variétés algébriques
(Org: **Xi Chen** (Alberta), **Steven Lu** (UQAM) and/et **Peter Russell** (McGill))

XI CHEN, University of Alberta
Rational self maps of Calabi-Yau manifolds

It is expected that a very general Calabi-Yau complete intersection in the projective space does not admit a dominant rational endomorphism of degree > 1. I will give a sketch of my proof of this statement and discuss some of its consequences.

CHARLES DORAN, University of Alberta
The 14th Case VHS and K3 Fibrations: Overview

We will discuss the construction and applications of Calabi-Yau threefolds admitting fibrations by certain lattice-polarized K3 surfaces of high Picard rank. The construction is motivated by a question of explicit geometric realization of a weight three variation of Hodge structure (VHS) on $\mathbb{P}^1 \setminus \{0, 1, \infty\}$ previously classified by Doran-Morgan.

GORDON HEIER, University of Houston
On curvature and the structure of projective Kaehler manifolds

We will discuss the structure of projective Kaehler manifolds based on various curvature assumptions. If the holomorphic sectional curvature is negative, we will present positivity theorems for the canonical line bundle. In the positive curvature case, positive total scalar curvature will be identified as a sufficient condition for uniruledness. This is joint work with S. S. Y. Lu and B. Wong.

JACQUES HURTUBISE, McGill UNiversity
Pseudo-real bundles on Kahler varieties

Let X be a compact connected Kähler manifold equipped with an anti-holomorphic involution which is compatible with the Kähler structure. Let G be a connected complex reductive affine algebraic group equipped with a real form σ_G. We define pseudo-real principal G–bundles on X; these are generalizations of real algebraic principal G–bundles over a real algebraic variety. Next we define stable, semistable and polystable pseudo-real principal G–bundles. Their relationships with the usual stable, semistable and polystable principal G–bundles are investigated. We then prove that the following Donaldson–Uhlenbeck–Yau type correspondence holds: a pseudo-real principal G–bundle admits a compatible Einstein-Hermitian connection if and only if it is polystable. A bijection between the following two sets is established: 1) The isomorphism classes of polystable pseudo-real principal G–bundles such that all the rational characteristic classes of the underlying topological principal G–bundle vanish. 2) The equivalence classes of twisted representations of the extended fundamental group of X in a σ_G–invariant maximal compact subgroup of G. (The twisted representations are defined using the central element in the definition of a pseudo-real principal G–bundle.) All these results are also generalized to the pseudo-real Higgs G–bundle. (with I. Biswas, O. Garcia-Prada)

JAMES LEWIS, University of Alberta
Hodge Type Conjectures and the Bloch-Kato Theorem

We will discuss a version of the Hodge conjecture for higher K-groups, and explore some consequences of an affirmative answer to this conjecture for Abel-Jacobi maps. We further explain the impact of the Bloch-Kato theorem on the cycle class map at the generic point, in the Milnor K-theory case. This talk is based on joint work with Rob de Jeu.

STEVEN LU, UQAM
Algebraic and analytic bubbles in algebraic varieties

In analysis, the failure in "compactness" in some space of maps to a compact analytic object X is often explained by the existence of "bubbles" in X. In holomorphic geometry (resp. algebraic geometry) bubbles are given by nonconstant holomorphic maps from $\mathbf{C}$, i.e. Brody curves, (resp. from $\mathbf{CP}^1$, i.e. rational curves) to X. We will discuss the role of bubbles for a quasi projective varieties U and their effect on the positivity of the log-canonical divisor of U. This is joint work with De-Qi Zhang.

DAVID MCKINNON, University of Waterloo
A Liouville Theorem for algebraic varieties

Liouville's Theorem provides a bound on how well rational numbers can approximate algebraic numbers. Roth's Theorem gives a much better bound in general, but for rational approximations of rational numbers, Liouville's Theorem is actually better, and is sharp to boot! In my talk, I will describe how Seshadri constants are involved in a generalization of Liouville's Theorem to rational approximations on arbitrary algebraic varieties, and deduce from all this a conjecture of McKinnon for certain cubic surfaces. This talk is based on joint work with Mike Roth of Queen's University.

ANDREY NOVOSELTSEV, University of Alberta
The 14th Case VHS and K3 Fibrations: Toric Approach

Following the talk by Charles Doran, we will discuss explicit toric construction of certain Calabi-Yau threefolds and their fibrations by lattice-polarized K3 surfaces of high Picard rank. Along the way we will demonstrate the use of toric capabilities of a free open-source mathematics software Sage.

GREGORY PEARLSTEIN, Michigan State University
The zero locus of the infinitesimal invariant

Let ν be a normal function on a complex manifold X. The infinitesimal invariant of ν has a well-defined zero locus inside the tangent bundle TX. When X is quasi-projective, and ν is admissible, we show that this zero locus is constructible in the Zariski topology.

ZIV RAN, UC Riverside
Differential Equations in Hilbert-Mumford Calculus

Hilbert-Mumford Calculus refers to the intersection theory of the tautological classes (Chern classes of tautological bundles) on the relative Hilbert scheme of a family of nodal curves. This Calculus is largely encoded in the powers of the discriminant polarization, and consequently can be encoded in a suitable generating function. We show this generating function satisfies a certain second-order differential equation of evolution type, which can be used to compute it.

SIMON ROSE, Queen's University
Counting hyperelliptic curves on abelian surfaces with quasi-modular forms

Let A be a polarized abelian surface. By a simple dimension count, we should expect a finite number of hyperelliptic curves in the class of the polarization, up to translation in A. In this talk we will go over a method to compute this, using Gromov-Witten theory, the crepant resolution conjecture, and the Yau-Zaslow formula.

MIKE ROTH, Queen's University
Seshadri constants, diophantine approximation, and Roth's theorem for arbitrary varieties

If X is a variety of general type defined over a number field k, then the Bombieri-Lang conjecture predicts that the k-rational points of X are not Zariski dense. One way to view the conjecture is that a global condition on the canonical bundle (that it is "generically positive") implies a global condition about rational points. By a well-established principle in geometry we should also look for local influence of positivity on the accumulation of rational points. To do that we need measures of both these local phenomena.

Let L be an ample line bundle on X, and $x \in X(\overline{k})$. By slightly modifying the usual definition of approximation exponent on $\mathbf{P}^1$, we define a new invariant $\alpha_x(L) \in (0, \infty]$ which measures how quickly rational points accumulate around x, as measured by L.

The central theme of the talk is the interrelations between $\alpha_x(L)$ and the Seshadri constant $\epsilon_x(L)$ which measures the local positivity of L near x. In particular, the classic approximation theorem of Klaus Roth on $\mathbf{P}^1$ generalizes as an inequality between α_x and ϵ_x valid for all projective varieties. This is joint work with David McKinnon (Waterloo).

PETER RUSSELL, McGill University
Forms of actions of the multiplicative group on affine 3-space

Let k be a field of characteristic 0. Let $\alpha : G \times X \to X$ be an effective action of an algebraic k-group G on an affine k-variety X that is a *k-form* of a (linear) $\mathbb{G}_m$-action on $\mathbb{A}^3$. (This means that for some field $K \supset k$ we have $G_K = \mathbb{G}_{m,K}, X_K = \mathbb{A}^3_K$ and α_K is linear. Note that for the first two conditions we can assume K/k finite and that the last then holds for $K = \overline{k}$.)
Theorem: $X \simeq \mathbb{A}^3$ and α is linearizable.
Corollary 1: A $\mathbb{G}_m$-action on $\mathbb{A}^3$ is linearizable.
Corollary 2: A k-form X of $\mathbb{A}^3$ that admits a non-trivial action of a reductive group is trivial.

ALAN THOMPSON, University of Alberta
The 14th Case VHS and K3 Fibrations: Elliptic Surfaces

We will discuss the construction and applications of Calabi-Yau threefolds admitting fibrations by certain lattice-polarized K3 surfaces of high Picard rank. This talk addresses a direct construction for these Calabi-Yau threefolds using elliptic surfaces.

YURI TSCHINKEL, Courant Institute, NYU
Del Pezzo surfaces over function fields of curves

The geometry of spaces of sections of Del Pezzo surface fibrations over curves is tightly linked with the global geometry of the total space of the fibration. I will report on joint work with B. Hassett establishing some basic results in this direction.

FRANKLIN VERA PACHECO, University of Toronto/Fields Institute
Resolving some singularities while preserving others.

Resolution of singularities consists in constructing a non-singular model of an algebraic variety. This is done by applying a proper birational map that is a local isomorphism at the smooth points. Often too much information is lost about the original variety if the smooth points are the only ones where the desingularization map is a local isomorphism. In these cases, a desingularization preserving some minimal singularities is necessary. This suggests the question of whether, given a class of singularity types $\mathcal{S}$, it is possible to remove with a birational map all singularities not in $\mathcal{S}$ while still having a local isomorphism over the singularities of type $\mathcal{S}$. We will talk about several instances of this problem and techniques that can be used to solve them.

Joint works with **Edward Bierstone, Sergio Da Silva, and Pierre Milman,** (University of Toronto/Fields Institute).

YANHONG YANG, Columbia University
Rationality of Euler-Chow series and finite generation of Cox rings

Let X be a smooth projective variety of dimension d over $\mathbb{C}$ with $\mathbf{Pic}(X) \simeq \mathbb{Z}^r$ for some r. It is a simple fact that finite generation of the Cox ring $\mathrm{Cox}(X)$ implies rationality of the Euler-Chow series $E_{d-1}(X)$. We discuss if the converse statement holds.

First, we construct a counterexample to the converse statement based on Hu-Keel's geometric characterization of varieties with finitely generated Cox rings.

Second, we prove that $E_{d-1}(X)$ is transcendental for the known rational spaces X with $\mathrm{Cox}(X)$ finitely generated and also deduce many more spaces with the properties from Mukai's examples, i.e. X is the blow-up of $(\mathbb{P}^{r-1})^{p-1}$ at $q+r$ points in general position, where $r > 2$ and $\frac{1}{p} + \frac{1}{r} + \frac{1}{q} \leq 1$.

Last, we deduce a recursive formula to compute $E_1(X)$ when X is a Del Pezzo surface and carry out the computation for X of degree 5.

This is joint work with Xi Chen (Alberta) and Javier Elizondo (Universidad Nacional Autónoma de México).

KIRILL ZAYNULLIN, University of Ottawa
The gamma-filtration on projective homogeneous spaces

Let X be a projective homogeneous variety of a simple linear algebraic group over an arbitrary field, e.g. a flag variety, an anisotropic quadric, a Severi-Brauer variety. In the present talk we discuss recent results on the torsion of the subsequent quotients of the Grothendieck gamma-filtration on X and its applications to the theory of torsors and cohomological invariants of linear algebraic groups.

TONG ZHANG, Department of Mathematics, University of Alberta
Relative Noether inequality on fibered surfaces

I will talk about a relative version of the classical Noether inequality for minimal surfaces of general type. I will also talk about some applications of it to the slope inequality for semistable families of curves and the Severi conjecture for surfaces of maximal Albanese dimension. It is a joint work with Xinyi Yuan.