Nonlinear Functional Differential Equations
Équations différentielles fonctionnelles non linéaires
(Org: **Hermann Brunner** (Memorial))

URI ASCHER, UBC
Stochastic algorithms for inverse problems involving PDEs and many measurements

Inverse problems involving systems of partial differential equations (PDEs) can be very expensive to solve numerically. This is so especially when many experiments, involving different combinations of sources and receivers, are employed in order to obtain reconstructions of acceptable quality. The mere evaluation of a misfit function (the distance between predicted and observed data) often requires hundreds and thousands of PDE solves. We develop and assess dimensionality reduction methods, both stochastic and deterministic, to reduce this computational burden.

We present in detail our methods for solving such inverse problems for the famous DC resistivity and EIT problems. These methods involve incorporation of a priori information such as piecewise smoothness, bounds on the sought conductivity surface, or even a piecewise constant solution. A more general random subset method is proposed first. We then assume that all experiments share the same set of receivers and concentrate on methods for reducing the number of combinations of experiments, called simultaneous sources, that are used at each stabilized Gauss-Newton iteration. Algorithms for controlling the number of such combined sources are proposed and justified. Evaluating the misfit approximately, except for the final verification for terminating the process, always involves random sampling. Methods for selecting the combined simultaneous sources, involving either random sampling or truncated SVD, are proposed and compared. Highly efficient variants of the resulting algorithms are identified.

JACQUES BÉLAIR, Université de Montréal
DISTRIBUTION OF DELAYS IN A PHARMACODYNAMIC MODEL

Time delays occur naturally in pharmacokinetic and pharmacodynamic (PK/PD) processes, but the form in which they are introduced in the models is not always entirely obvious, the distribution of delays being typically ignored or represented empirically. We present a model of chemotherapy-induced myelosuppression using differential equations with distributed delays, to take into account the delay between administration of the drug and the observed effect. The transit compartment yields a single differential equation with a bimodal distribution of delays. We discuss the stability of this system, obtaining a stability chart in a two parameter space, and possible oscillatory solutions.

Joint work with Andreea Rimbu Pruncut.

SUE ANN CAMPBELL, University of Waterloo
Plankton Models with Time Delay

We consider a three compartment (nutrient-phytoplankton-zooplankton) model with nutrient recycling. When there is no time delay the model has a conservation law and may be reduced to an equivalent two dimensional model. We consider how the conservation law is affected by the presence of time delay (both discrete and distributed) in the model. We study the stability and bifurcations of equilibria when the total nurient in the system is used as the bifurcation parameter. This is joint work with Matt Kloosterman and Francis Poulin.

TONY HUMPHRIES, McGill University
Stability and numerical stability of a model state-dependent DDE

We consider the stability properties of the model state-dependent delay differential equation (DDE),

$$\dot{u}(t) = \mu u(t) + \sigma u(t - a - cu(t)),$$

and its numerical discretization by the backward Euler method. The stability region for the DDE itself is well-known in the constant delay case ($c = 0$), and is the basis for several numerical stability definitions. Recent results in state-dependent DDEs show that (with the possible exception of points on the boundary) the state-dependent DDE ($c \neq 0$) has the same stability region as the constant delay DDE. We thus propose this as a model problem in the numerical analysis of state-dependent DDEs, and extend the numerical stability definitions accordingly.

To study the stability properties of the backward Euler method, we first study the DDE itself and use a Lyapunov-Razumikhin approach to directly prove stability of the state-dependent DDE in parts of its stability region including the entire delay-independent portion and parts of the delay-dependent portion. The Lyapunov-Razumikhin approach is further generalised to study the stability of the backward Euler method solution, and we establish stability for all step-sizes in the part of the stability region for which the direct proof showed stability of the DDE, and stability for sufficiently large step-size in the entire stability region.

Joint work with Felicia Magpantay (York) & Nicola Guglielmi (L'Aquila)

MICHAEL MACKEY, McGill University
Modeling chemotherapy effects on hematopoiesis

The delivery of chemotherapy has, almost without exception, profound side effects on many physiological systems and the regulation of hematopoiesis is not an exception. In this talk I will outline the work that is ongoing in our attempts to minimize the hematopoietic side effects of chemotherapy. This modeling work is naturally framed within the context of functional differential equations with state dependent delays, and has revealed some of the many potential dynamical effects of forcing systems so described with external perturbations like those due to chemotherapy. The dynamic effects that the numerical solutions of these equations have revealed offer important clues about how clinicians may be able to avoid some of the side effects of chemotherapy. Specifically we find that the mathematical model for the regulation of hematopoiesis shows significant resonance effects at certain periods of chemotherapy administration that are probably associated with especially adverse reactions in patients.

FELICIA MAGPANTAY, York University
An age-structured population model with state-dependent delay: derivation and numerical integration

We present an age-structured population model that accounts for the following aspects of complex life cycles: (1) There are juvenile and adult stages, (2) only the adult stage is capable of reproducing, (3) cohorts of juveniles can transition to the adult stage when they have consumed enough nutrition and (4) the juvenile and adult populations consume different limited food sources. Taking all of these into account leads to a new mathematical model that cannot be directly analyzed using the established framework of functional differential equations. The model consists of a partial differential equation with a nonlinear boundary condition and state-dependent delay due to a threshold condition. In this talk we present the derivation of the model, its properties and a numerical scheme to integrate the equations.

This is joint work with Nemanja Kosovalic and Jianhong Wu.

CHUNHUA OU, Memorial University
Delayed reaction diffusion models and traveling waves for spatial spread of rabies in Europe: a re-visit

In this talk, spatial spread of rabies in Europe is re-visited with the consideration of the impacts of the incubation and its interaction with the spatial movement of the susceptible and the incubative. First, a delayed reaction diffusion model is constructed with the incorporation of the incubation only. The minimal spreading speed is derived by the classical stability analysis and it is shown to be a decreasing function of the incubation time. A new method based on the integration of wave pulses is proposed which yields the existence of wave patterns for all values of the incubation time. In addition, by incorporating the spatial movement of the incubative foxes, a non-local reaction diffusion system is constructed. Rigorous proof of the existence of the wave patterns is shown via the integration of the wave pulses coupled with the Fredholm alternative theorem.

GAIL WOLKOWICZ, McMaster University
Dynamics of the chemostat and classical predator-prey models with time delay

The dynamics of the classical predator-prey model and the predator-prey model based in the chemostat are studied and compared to see whether a discrete time delay in the conversion process can lead to sustained oscillatory behaviour, when no such behaviour is possible when delay is ignored. A surprising similarity between the possible attractors of the classical predator-prey model and the attractors of the Mackey-Glass equation are demonstrated. The analogous integro-differential equations models are also considered.

JIANHONG WU, York University
Global dynamics of the Nicholson's blowflies equation revisited: onset and termination of nonlinear oscillations

We revisit the Nicholson's blowflies model with natural death rate incorporated into the delay feedback. We consider the delay as a bifurcation parameter and examine the onset and termination of Hopf bifurcations of periodic solutions from a positive equilibrium. We show that the model has only a finite number of Hopf bifurcation values and we describe how branches of Hope bifurcations are paired so the existence of periodic solutions with specific oscillation frequencies occurs only in bounded delay intervals. The bifurcation analysis and the Matlab package DDE-BIFTOOL developed by Engelborghs et al guide some numerical simulations to identify ranges of parameters for coexisting multiple attractive periodic solutions. This is a joint work with Hongying Shu and Lin Wang.

YUAN YUAN, Memorial University of newfoundland
Stability and Bifurcation in FDE with Distributed Delay

A set of sufficient conditions for the global and local stability is established for a large class of functional differential equation with distributed delay. With the loss of the stability at the boundary of the stability regions, we discuss the Hopf bifurcation, there the computation of the coefficients are given in the form of the corresponding characteristic equation explicitly. Then these analytic results are applied to the mathematical models of white blood cell production. Numerical simulations are presented to illustrate the stability regions of parameters and to address the effect of the distributed time delay in the physiological oscillations.

This talk is based on my joint work with Drs. Jacques Belair and Xiaoqiang Zhao.

XIAOQIANG ZHAO, Memorial University of Newfoundland
Global Dynamics of A Time-Delayed Reaction and Diffusion Malaria Model

In this talk, I will report our recent research on a vector-bias malaria model with incubation period and diffusion. We first prove the global stability of the disease-free or endemic equilibrium for the spatially homogeneous time-delayed system. Then we establish the threshold dynamics for the spatially heterogeneous system in terms of the basic reproduction ratio. We also obtain a set of sufficient conditions for the global attractivity of the positive steady state. This talk is based on my joint work with Zhiting Xu.

XINGFU ZOU, University of Western Ontario
On a DDE model describing malaria transmission dynamics in a patch environment

I will present some results on a DDE model that describes the transmission dynamics of malaria over a patchy environment. The model incorporates two important factors into the classic Ross-McDonand model: disease latencies in both humans and mosquitoes, and dispersal of humans between patches. The basic reproduction number $\mathcal{R}_0$ of model is identified by the theory of the next generation operator for structured disease models and the dynamics of the model is investigated in terms of $\mathcal{R}_0$. It is shown that the disease free equilibrium is asymptotically stable if $\mathcal{R}_0 < 1$, and it is unstable if $\mathcal{R}_0 > 1$; in the latter case, the disease is endemic in the sense that the variables for the infected compartments are uniformly persistent. For the case of two patches, more explicit formulas for $\mathcal{R}_0$ are derived by which, impacts of the dispersal rates as well as the latency delays on disease dynamics are explored. Some numerical computations for $\mathcal{R}_0$ in terms of dispersal rates are carried out, which visually show that the impacts could be very complicated: in certain range of the parameters, $\mathcal{R}_0$ is increasing with respect to a dispersal rate while in some other range, it can be decreasing with respect to the same dispersal rate.

This is a joint work with Dr. Yanyu Xiao.