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Stability and numerical stability of a model state-dependent DDE

We consider the stability properties of the model state-dependent delay differential equation (DDE),

$$\dot{u}(t) = \mu u(t) + \sigma u(t - a - cu(t)),$$

and its numerical discretization by the backward Euler method. The stability region for the DDE itself is well-known in the constant delay case ($c = 0$), and is the basis for several numerical stability definitions. Recent results in state-dependent DDEs show that (with the possible exception of points on the boundary) the state-dependent DDE ($c \neq 0$) has the same stability region as the constant delay DDE. We thus propose this as a model problem in the numerical analysis of state-dependent DDEs, and extend the numerical stability definitions accordingly.

To study the stability properties of the backward Euler method, we first study the DDE itself and use a Lyapunov-Razumikhin approach to directly prove stability of the state-dependent DDE in parts of its stability region including the entire delay-independent portion and parts of the delay-dependent portion. The Lyapunov-Razumikhin approach is further generalised to study the stability of the backward Euler method solution, and we establish stability for all step-sizes in the part of the stability region for which the direct proof showed stability of the DDE, and stability for sufficiently large step-size in the entire stability region.

Joint work with Felicia Magpantay (York) & Nicola Guglielmi (L'Aquila)