IBRAHIMA DIONE, Université Laval

Penalty/finite element approximations of slip boundary conditions and Babuska's paradox

The penalty method is a classical and widespread method for the numerical treatment of constrained problems such as unilateral contact problems and problems with Dirichlet boundary conditions. It provides an alternative approach to constrained optimization problems which avoids the necessity of introducing additional unknowns in the form of Lagrange multipliers. In the case of slip boundary conditions for fluid flows or elastic deformations, one of the main obstacle to their efficiency and to their mathematical analysis is that a Babuska's type paradox occurs.

Observed first by Sapondzyan [2] and Babuska [1] on the plate equation in a disk with simple support boundary conditions, Babuska's paradox can be stated as follows: on a sequence of polygonal domains converging to the domain with a smooth boundary, the solutions of the corresponding problems do not converge to the solution of the problem on the limit domain.

Our presentation will focus on the finite element approximation of Stokes equations with slip boundary conditions imposed with the penalty method in two and three space dimensions. For a polygonal or polyhedral boundary, we prove convergence estimates in terms of both the penalty and discretization parameters. In the case of a smooth curved boundary, we show through a numerical example that convergence may not hold due to a Babuska'type paradox. Finally, we propose and test numerically several remedies.

[1] Babuska I. and Pitkaranta J. SIAM J. MATH. ANAL, 21 (1990) 551-576.

[2] Sapondzyan O.M. Akad. Nauk Armyan. SSR. Izv. Fiz.-Mat. Estest. Tehn. Nauki, 5:29–46, 1952.