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A Brief Introduction to Expanders and Ramanujan Graphs

Think of a graph as a communications network. Putting in edges (e.g., fiber optic cables, telephone lines) is expensive, so we wish to limit the number of edges in the graph. At the same time, we would like the communications network to be as fast and reliable as possible. We will see that the quality of the network is closely related to the eigenvalues of the graph's adjacency matrix. Essentially, the smaller the eigenvalues are, the better the communications network is. It turns out that there is a bound, due to Alon, Serre, and others, on how small the eigenvalues can be. This gives us a rough sense of what it means for graphs to represent "optimal" communications networks; we call these Ramanujan graphs. Families of k-regular Ramanujan graphs have been constructed in this manner by Lubotzky, Sarnak, and others whenever k-1 equals a power of a prime number. No one knows whether families of k-regular Ramanujan graphs exist for all k.