Celestial Mechanics
Mécanique céleste

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LENNARD BAKKER, Brigham Young University

The Rhomboidal Symmetric-Mass Four-Body Problem.

We consider the existence and stability of periodic solutions with regularizable collisions in the rhomboidal symmetric-mass four-body problem. In the two degrees of freedom setting, where the analytic existence of the periodic solutions is given by a variational method, we show that the periodic solutions are numerically linearly stable for most of the values of the mass parameter. In the four degrees of freedom setting, we establish the analytic existence of the periodic solutions and numerically investigate their linear stability.

FLORIN DIACU, University of Victoria

Rotopulsators of the curved N-body problem

We consider the N-body problem in spaces of constant curvature and study its rotopulsators, i.e. solutions for which the configuration of the bodies rotates and, usually, changes size during the motion. Rotopulsators fall naturally into five groups: positive elliptic, positive elliptic-elliptic, negative elliptic, negative hyperbolic, and negative elliptic-hyperbolic, depending on the nature and number of their rotations and on whether they occur in spaces of positive or negative curvature. After obtaining existence criteria for each type of rotopulsator, we derive their conservation laws. We further deal with the existence and uniqueness of some classes of rotopulsators in the 3-body case and prove two general results about the qualitative behaviour of these orbits. An interesting finding is that of a class of rotopulsators that behave like relative equilibria, i.e. maintain constant mutual distances during the motion, but cannot be generated from any single element of the underlying subgroup $SO(2) \times SO(2)$ of the Lie group $SO(4)$.

TOSHIAKI FUJIWARA, Kitasato University

Saari's homographic conjecture for the planar equal-mass three-body problem under the Newton potential and a strong force potent

The Saari's homographic conjecture for N-body problem is the following: For the N-body problem under the homogenious potential $U = \sum m_i m_j / r_{ij}^{\alpha}$, the configurational measure $I^{\alpha/2}U$ is constant if and only if the motion is homographic. Here, m_i $(i = 1, 2, \ldots N)$ is mass for the body i, r_{ij} is the mutual distance between the body i and j and $I = \sum m_i m_j r_{ij}^2$ is the moment of inertia.

In this year, Fukuda, Ozaki, Taniguchi and the present author proved this conjecture for the planar equal-mass three-body problem under the Newton potential($\alpha = 1$) and the strong force potential($\alpha = 2$). In this talk, I will review our work.

This is a joint work with Hiroshi Fukuda, Hiroshi Ozaki and Tetsuya Taniguchi.

JUAN LUIS GARCIA GUIRAO, Universidad Politécnica de Cartagena, SPAIN

Periodic orbits and $\mathcal{C}^1$-integrability in the planar Stark-Zeeman problem

The aim of the present talk is to study the periodic orbits of a hydrogen atom under the effects of a circularly polarized microwave field and a static magnetic field orthogonal to the plane of polarization of the magnetic field via averaging theory. Moreover, the technique used for proving the existence of isolated periodic orbits allows us to provide information on the $\mathcal{C}^1$–integrability of this mechanic–chemical system.

JEFFREY LAWSON, Western Carolina University
Saari's Conjecture on simple mechanical systems with symmetry

In 1970 Donald Saari famously conjectured that if a Newtonian N-body system has a constant moment of inertia then it is in relative equilibrium (i.e., it is in rigid rotation with a constant angular velocity). In 2002 Jerrold Marsden hypothesized that the conjecture may be generalized to a simple mechanical system that admits a Lie symmetry. The aim of this talk is to explore a geometric approach to Saari's Conjecture. In particular we may pose the problem on a Manakov rigid body and prove a refined statement of the conjecture. (The refinement is essentially necessary to handle higher dimensional symmetry.) By employing a Palais slice decomposition, the question may be further enlarged to simple mechanical systems in which the group action has no points of isotropy. We will conclude with a brief discussion on handling points of isotropy by using a blowup technique.

ABDALLA MANSUR, Queen's University
The Maslov index and Stability of Periodic Solutions

We employ techniques from symplectic geometry and specifically a variant of the Maslov index for curves of Lagrange planes along action-minimizing solutions to develop conditions which preclude eigenvalues of the monodromy matrix on the unit circle. This analytical method of proving stability is demonstrated in the context of selected special cases of the n-body problem, namely rhombus and parellelogram solutions of the four-body problem and hip-hop solutions of the 2n-body problem.

DANIEL OFFIN, Queen's University
Dynamics, stability and symmetric minimization

We discuss the connection between minimization of action and hyperbolic structure of invariant sets. The notion of absolute minimization always leads to hyperbolic behavior. The weaker notion of symmetric minimization over fundamental time domain, or equivalently minimization over a symmetry class may involve hyperbolic behavior. We give examples from N-body dynamics and outline the proof of hyberbolicity in the reduced space for the periodic hip hop family of the 2N-body problem with equal mass. Finally we state a condition for periodic symmetric minimizing orbits to be absolutely minimizing over arbitrary compact time intervals. This condition concerns the representation of the reversing subgroup of the spatio-temporal group for the given periodic solution.

GARETH ROBERTS, College of the Holy Cross
Stability of Relative Equilibria in the N-Vortex Problem

In the weather research and forecasting models of certain hurricanes, "vortex crystals" are found within a polygonal-shaped eyewall. These special configurations can be interpreted as relative equilibria (rigidly rotating solutions) of the point vortex problem introduced by Helmholtz. Their stability is thus of considerable importance. Adapting an approach of Moeckel's for the companion problem in celestial mechanics, we present some useful theory for studying the linear stability of relative equilibria in the N-vortex problem. The analysis is developed in a rotating coordinate frame and special properties of the Hamiltonian play a key role. For example, we show that in the case of equal strength vortices, a relative equilibrium is linearly stable if and only if it is a minimum of the Hamiltonian restricted to a level surface of the moment of inertia. Some symmetric examples will be presented, including a linearly stable family of rhombii in the four-vortex problem.

MANUELE SANTOPRETE, Wilfrid Laurier University
Relative equilibria in the four-vortex problem with two pairs of equal vorticities

The N-vortex problem concerns the dynamics of N point vortices moving in the plane. Of particular interest in this problem are solutions that appear fixed when viewed in an uniformly rotating frame. Such solutions are called relative equilibria. In this talk I will give a complete classification of the relative equilibria in the four vortex problem with two pairs of equal vorticities. I will also describe some of the methods used to obtain such results.

CRISTINA STOICA, Wilfrid Laurier University
Relative equilibria in symmetric $(2N+1)$-body problems

Consider the Newtonian $(2N+1)$-body problem where $2N$ of the bodies have unit mass and at any time form two regular N-gons with a common centre, and where an additional mass m is centrally situated. It is known that in this context there is a $m_0 > 0$ so that the number of central configurations is three for $m < m_0$ and one if $m > m_0$. Also, it is known that there is a $m_c > 0$, $m_c \neq m_0$, so that the regular $2N$-gon with central mass is linearly stable if $m > m_c$.

Using the discrete and rotational symmetries, we reduce the problem to a three degrees of freedom Hamiltonian system. In this setting, we show that the central configurations mentioned above are in fact relative equilibria and that m_0 marks a pitchfork/steady-state bifurcation. The value m_c marks a Hamiltonian-Hopf bifurcation (i.e., a 1:-1 resonance).

J. GUADALUPE REYES VICTORIA, UAM