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Diophantine approximation with sign constraints

Let a and b be real numbers such that $1, a$ and b are linearly independent over \mathbb{Q} . A classical result of Dirichlet asserts that there are infinitely many triples of integers (x, y, z) such that $|ax + by + z| < \max(|x|, |y|, |z|)^{-2}$. In 1976, W. M. Schmidt asked what can be said under the restriction that x and y be positive. Upon denoting by $\gamma \cong 1.618$ the golden ratio, he proved that there are triples $(x, y, z) \in \mathbb{Z}^3$ with $x, y > 0$ for which the product $|ax + by + z| \max(|x|, |y|, |z|)^\gamma$ is arbitrarily small. Although, at that time, Schmidt did not rule out the possibility that γ could be replaced by any number smaller than 2, N. Moshchevitin proved this year that it cannot be replaced by a number larger than 1.947. In this talk, we present a construction showing that the result of Schmidt is in fact optimal.