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Rigorous Computations for Infinite Dimensional Dynamical Systems

Studying and proving the existence of solutions of nonlinear dynamical systems using standard analytic techniques is a challenging problem. In particular, this problem is even more challenging for partial differential equations, variational problems or functional delay equations which are naturally defined on infinite dimensional function spaces. As a consequence of these challenges and with the recent availability of powerful computers and sophisticated software, numerical simulations quickly became one of the primary tool used by scientists to conjecture the behaviour of the dynamics of the above mentioned nonlinear equations. A standard approach adopted by mathematicians is to get insights from numerical simulations to formulate new conjectures, and then attempt to prove the conjectures using pure mathematical techniques only. As one shall argue, this strong dichotomy need not exist in the context of dynamical systems, as the strength of numerical analysis and functional analysis can be combined to prove, in a direct computational way, existence of solutions of infinite dimensional dynamical systems. The goal of this talk is to present such rigorous numerical methods to the context of proving the existence of steady states, time periodic solutions, traveling waves and connecting orbits of finite and infinite dimensional differential equations.