Partial Differential Equations and Spectral Theory
Équations aux dérivées partielles et théorie spectrale

(Org: **Dmitry Jakobson** (McGill) and/et **Iosif Polterovich** (Montréal))

ALMUT BURCHARD, University of Toronto

Perimeter under multiple Steiner symmetrizations

Steiner symmetrization along n linearly independent directions in n-space transforms every compact set into a set of finite perimeter.

(Joint work with Gregory R. Chambers.)

YAIZA CANZANI, McGill University

Distribution of randomly propagated Schrödinger eigenfunctions

This is joint work with Dmitry Jakobson and John Toth. Let (M, g_0) be a compact Riemmanian manifold, and $V \in C^\infty(M)$. Let $P_0(h) := -h^2\Delta_{g_0} + V$, be the semiclassical Schrödinger operator for $h \in (0, h_0]$. If φ_h is an L^2-normalized eigenfunction of $P_0(h)$, then $\int_A |\varphi_h(x)|^2 dv_{g_0}(x)$ is interpreted as the probability that a quantum particle of energy $\sim 1/h^2$ belongs to $A \subset M$. For a quantum particle with initial state φ_h, its evolution at time t is described by the same probabilty density since $|e^{-\frac{it}{h}P_0(h)}\varphi_h| = |\varphi_h|$. However, since real life systems are usually affected by "noise", the time evolution is better decribed by the state

$$\varphi_h^{(u)}(x) = e^{-\frac{it}{h}P_u(h)}\varphi_h$$

where $P_u(h)$ is some small perturbation of $P_0(h)$.

In this talk we consider a smooth family of perturbations g_u of the reference metric g_0 for $u \in \mathcal{B}^k(\varepsilon) \subset \mathbb{R}^k$ of radius $\varepsilon > 0$, and consider the perturbed Schrödinger operators $P_u(h) := -h^2\Delta_{g_u} + V$. For $t > 0$ small, we study the moments of the real part of the perturbed eigenfunctions regarded as random variables

$$Re\left(\varphi_h^{(\cdot)}(x)\right) : \mathcal{B}^k(\varepsilon) \to \mathbb{R} \qquad \text{for } x \in M.$$

LAYAN EL HAJJ, McGill

Intersection bounds for planar Neumann eigenfunctions with interior analytic curves

Let $\Omega \subset \mathbb{R}^2$ be a bounded piecewise smooth domain and ϕ_λ be a Neumann (or Dirichlet) eigenfunction with eigenvalue λ^2 and nodal set $\mathcal{N}_{\phi_\lambda} = \{x \in \Omega; \phi_\lambda(x) = 0\}$. Let $H \subset \Omega$ be an interior C^ω curve. Consider the intersection number

$$n(\lambda, H) := \#(H \cap \mathcal{N}_{\phi_\lambda}).$$

We first prove that for general piecewise-analytic domains, and under an appropriate "goodness" condition on H ,

$$n(\lambda, H) = \mathcal{O}_H(\lambda)\ (*)$$

as $\lambda \to \infty$. We then prove that the bound in $(*)$ is satisfied in the case of quantum ergodic (QE) sequences of interior eigenfunctions, provided Ω is convex and H has strictly positive geodesic curvature.

SURESH ESWARATHASAN, McGill University/CRM

Lower bounds for the Weyl Remainder

This talk will address omega bounds for the remainder term in Weyl's Law for piecewise C^∞ domains. This is joint work with Iosif Polterovich and John Toth.

RENJIE FENG, CRM and Mcgill Unviersity
Some recent results on Gaussian random fields over Kahler manifolds

I will present some results on Gaussian Random homomorphic sections over Kahler manifolds which is a generalization of Kac's polynomials to Kahler manifolds. The distribution of zeros is studied by Bleher-Shiffman-Zelditch; critical points by Douglas-Shiffman-Zelditch. Recently, I studied the critical values on SU(2) random polynomials with Z. Wang; I also studied critical values of random holomorphic sections on compact Kahler manifolds under the both Gaussian and Spherical ensembles with S.Zelditch.

WENYING FENG, Trent University
Nonlinear spectra and related maps: generalizations and applications

I will introduce the nonlinear spectrum defined by three European mathematicians (Furi, Martelli and Vignoli), and its further generalizations in different directions. Classes of nonlinear maps related to the nonlinear spectra such as stably solvable operators, 0-epi maps, (a, q)-stably solvable operators, L-stably solvable operators, their properties and applications will be discussed. I will also present some new results on nonlinear operators that can be decomposed into a linear operator and a nonlinear map. Applications to existence of solutions for some boundary value problems are obtained.

NASSIF GHOUSSOUB, University of British Columbia
Stability and Regularity in elliptic systems and 4th order equations

We examine nonlinear fourth order eigenvalue problems on bounded domains of N-dimensional space. We show –among other things– that for exponential nonlinearities, the extremal solution is smooth provided the dimension N is below 10.718. To do that, we isolate a new stability inequality satisfied by minimal solutions that is more amenable to estimates, as it allows a method of proof reminiscent of the second order case. This new approach leads to substantial improvements of various results on critical dimensions obtained recently by various authors. This is joint work with Craig Cowan.

NIKY KAMRAN, McGill University
Local energy decay for Dirac fields in the 5-dimensional Myers-Perry black hole geometry

We consider massive Dirac fields evolving in the exterior region of a 5-dimensional Myers-Perry black hole. Our main result states that the local energy of such fields decays in a weak sense at late times. This is proved in two steps. First, using the separability of the Dirac equation, we prove the absence of a pure point spectrum for the corresponding Dirac operator. Second, using a new form of the equation adapted to the two rotations axes of the black hole, we show by a Mourre theory argument that the spectrum is absolutely continuous. The result then follows. This is joint work with Thierry Daude (Cergy-Pontoise).

ALEXEY KOKOTOV, Concordia
Krein formula and S-matrix for Euclidean Surfaces with Conical Singularities

We use the Krein formula and the S-matrix formalism to give formulas for the zeta-regularized determinant of non-Friedrichs extensions of the Laplacian on Euclidean surfaces with conical singularities. This formula involves S(0) and we show that the latter can be expressed using the Bergman projective connection on the underlying Riemann surface. The talk is based on the joint work with Luc Hillairet (Nantes).

BENJAMIN LANDON, McGill University
The scattering length at positive temperature

A positive temperature analogue of the scattering length of a potential V can be defined via integrating the difference of the heat kernels of $-\Delta$ and $-\Delta + V$, with $-\Delta$ the Laplacian. An upper bound on this quantity is a crucial input in the derivation of a bound on the critical temperature of a dilute Bose gas, obtained in 2009 by R. Seiringer and D. Ueltschi. This bound on the critical temperature was given in the case of finite range potentials and sufficiently low temperature. In this paper, we improve the bound on the scattering length and extend it to potentials of infinite range. (Joint work with R. Seiringer)

ARIAN NOVRUZI, University of Ottawa
About the regularity of optimal convex domains

We will discuss the issue of the regularity of a domain, minimizing a shape functional under the convexity constraint, in dimension two and higher. Joint work with J. Lamboley and M. Pierre.

FRÉDÉRIC ROCHON, UQAM
Compactness of relatively isospectral sets of surfaces with cusps

We introduce a notion of relative isospectrality for surfaces with boundary having possibly non-compact ends either conformally compact or asymptotic to cusps. We obtain a compactness result for such families via a conformal surgery that allows us to reduce to the case of surfaces hyperbolic near infinity recently studied by Borthwick and Perry, or to the closed case by Osgood, Phillips and Sarnak if there are only cusps. This is a joint work with Pierre Albin and Clara Aldana.

ROBERT SEIRINGER, McGill University
A positive density analogue of the Lieb-Thirring inequality

The Lieb-Thirring inequalities give a bound on the negative eigenvalues of a Schroedinger operator in terms of an L^p norm of the potential. This is dual to a bound on the H^1-norms of a system of orthonormal functions. Here we extend these to analogous inequalities for perturbations of the Fermi sea of non-interacting particles, i.e., for perturbations of the continuous spectrum of the Laplacian by local potentials. (This is joint work with R. Frank, M. Lewin and E. Lieb.)

DAVID SHER, McGill University/CRM
The determinant on flat conic surfaces with excision of disks

Let (M, g) be a surface with a flat conical metric, and consider the surface M_ϵ obtained by removing disks of radius ϵ around any subset of the conical singularities. We investigate the asymptotic behavior of the determinant of the Laplacian on M_ϵ, with Dirichlet boundary conditions, as ϵ approaches zero. Such families of surfaces were first studied by Khuri in the context of the Osgood-Phillips-Sarnak approach to compactness of isospectral sets of metrics. In particular, Khuri used these surfaces to prove that the determinant is not a proper map on the moduli space of flat metrics on a surface of genus p with n disks removed when $np \geq 1$. Our goal is to analyze the determinant of the Laplacian more closely as a function on this moduli space. As a first step, we sharpen and generalize Khuri's results; our main formula is an asymptotic expansion for the determinant of the Laplacian on M_ϵ, with an error which vanishes as ϵ approaches zero. The proof uses determinant gluing formulas as well as an extension of an argument of Wentworth, which enables us to analyze the asymptotics of the Dirichlet-to-Neumann operator on M_ϵ.

VITALI VOUGALTER, University of Cape Town
Existence and nonlinear stability of stationary states for the semi-relativistic Schroedinger-Poisson system

We study the stationary states of the semi-relativistic Schroedinger-Poisson system in the repulsive (plasma physics) Coulomb case. In particular, we establish the existence and the nonlinear stability of a wide class of stationary states by means of the energy-Casimir method. Moreover, we establish global well-posedness results for the semi-relativistic Schroedinger-Poisson system in appropriate functional spaces.