A novel symmetry method is used to obtain exact solutions to Schrodinger equations with a power nonlinearity in multi-dimensions. The method uses a separation technique to solve an equivalent first-order group foliation system whose independent and dependent variables consist of the invariants and differential invariants of the point symmetry generators admitted by the Schrodinger equation. Many explicit new solutions are obtained which have interesting analytical behavior connected with blow-up and dispersion. These solutions include new similarity solutions and other new group-invariant solutions, as well as new solutions that are not invariant under any point symmetries of the Schrodinger equation. In contrast, standard symmetry reduction leads to nonlinear ODEs for which few if any explicit solutions can be derived by familiar integration methods.