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Rationality of Euler-Chow series and finite generation of Cox rings

Let X be a smooth projective variety of dimension d over $\mathbb{C}$ with $\mathbf{Pic}(X) \simeq \mathbb{Z}^r$ for some r. It is a simple fact that finite generation of the Cox ring $\mathrm{Cox}(X)$ implies rationality of the Euler-Chow series $E_{d-1}(X)$. We discuss if the converse statement holds.

First, we construct a counterexample to the converse statement based on Hu-Keel's geometric characterization of varieties with finitely generated Cox rings.

Second, we prove that $E_{d-1}(X)$ is transcendental for the known rational spaces X with $\mathrm{Cox}(X)$ finitely generated and also deduce many more spaces with the properties from Mukai's examples, i.e. X is the blow-up of $(\mathbb{P}^{r-1})^{p-1}$ at $q + r$ points in general position, where $r > 2$ and $\frac{1}{p} + \frac{1}{r} + \frac{1}{q} \leq 1$.

Last, we deduce a recursive formula to compute $E_1(X)$ when X is a Del Pezzo surface and carry out the computation for X of degree 5.

This is joint work with Xi Chen (Alberta) and Javier Elizondo (Universidad Nacional Autónoma de México).