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Forms of actions of the multiplicative group on affine 3-space

Let k be a field of characteristic 0. Let $\alpha : G \times X \to X$ be an effective action of an algebraic k-group G on an affine k-variety X that is a k-*form* of a (linear) $\mathbb{G}_m$-action on $\mathbb{A}^3$. (This means that for some field $K \supset k$ we have $G_K = \mathbb{G}_{m,K}, X_K = \mathbb{A}^3_K$ and α_K is linear. Note that for the first two conditions we can assume K/k finite and that the last then holds for $K = \overline{k}$.)
Theorem: $X \simeq \mathbb{A}^3$ and α is linearizable.
Corollary 1: A $\mathbb{G}_m$-action on $\mathbb{A}^3$ is linearizable.
Corollary 2: A k-form X of $\mathbb{A}^3$ that admits a non-trivial action of a reductive group is trivial.