Set Theory
Théorie des ensembles
(Org: **Ilijas Farah** (York))

SAMUEL COSKEY, York University and The Fields Institute
Cardinal invariants and the Borel Tukey order

Many proofs of inequalities between cardinal characteristics of the continuum are combinatorial in nature. These arguments can be carried out in any model of set theory, even a model of CH where the inequalities themselves are trivial. Thus, such arguments appear to establish a stronger relationship than a mere inequality. The Borel Tukey order was introduced by Blass in a 1996 article to address just this. Specifically, he observed that the combinatorial information linking two cardinal characteristics is often captured by a pair of Borel maps called a ¡em¿Borel Tukey morphism¡/em¿. The existence of a Borel Tukey morphisms between two cardinal invariants has since been found to have a couple of applications in other set-theoretic contexts. In this talk we will discuss a number of popular combinatorial cardinal invariants, and compare their traditional ordering of provable inequalities with the finer ordering given by the Borel Tukey morphisms.

SEAN COX, Universität Münster
Catching antichains

The notion of *antichain catching* appeared in the Foreman-Magidor-Shelah paper on Martin's Maximum, and was used extensively in Woodin's proofs of the presaturation of various stationary tower forcings. For normal ideals $\mathcal{I}$ and $\mathcal{J}$, let us say that $\mathcal{J}$ *catches* $\mathcal{I}$ (and write $catch(\mathcal{J},\mathcal{I})$) iff $\mathcal{J}$ has sufficiently large support, the $\mathcal{J}$-positive sets project onto the $\mathcal{I}$-positive sets in a certain canonical manner (as ideals), and whenever $G \subset (\mathcal{J}^+, \subset)$ is generic then the projection of G is generic for $(\mathcal{I}^+, \subset)$. Certain instances of $catch(\mathcal{J},\mathcal{I})$ are equivalent to saturation of $\mathcal{I}$ (namely when $\mathcal{J}$ is the *conditional club filter relative to* $\mathcal{I}$; see Foreman's chapter in Handbook of Set Theory). But in general the statement:

"there exists a $\mathcal{J}$ such that $catch(\mathcal{J},\mathcal{I})$"

is strictly weaker than saturation of $\mathcal{I}$ and strictly stronger than precipitousness of $\mathcal{I}$. I will discuss this result and others from some joint work with Martin Zeman; I will also discuss some joint work with Matteo Viale which made use of related notions.

JAMES CUMMINGS, Carnegie Mellon University
The number of normal measures

(joint work with Arthur Apter and Katherine Thompson) We discuss some results on the set of normal measures on a measurable cardinal, some in ZFC and some in a choiceless setting.

JOHN KRUEGER, University of North Texas
Failure of Square Properties

Square principles, which were introduced by Jensen in the context of the constructible universe L, are combinatorial properties which witness the uniform structure of canonical inner models. The construction of models in which square properties fail is an active and deep area of forcing and consistency results. We survey some of the more recent developments in this area, including the failure of partial square and good scales.

ALEKSANDRA KWIATKOWSKA, University of Illinois at Urbana-Champaign
Ample generics and the group of homeomorphisms of the Cantor set

A topological group G has ample generics if it has a comeager conjugacy class in every dimension, that is, if for every m the diagonal conjugacy action of G on G^m has a comeager orbit. We discuss several examples and properties of groups with ample generics. Then, answering a question of Kechris and Rosendal, we show that the group of all homeomorphisms of the Cantor set has ample generics. The main tool we use is the projective Fraisse theory developed by Irwin and Solecki.

CLAUDE LAFLAMME, University of Calgary
Groups containing the automorphism group of the Rado graph

We discuss groups of permutations containing the automorphism group of the Rado graph.

S. Thomas determined there are exactly five reducts (the closed ones), and P. Cameron and S. Tarzi investigated other natural overgroups. We review their results and answer some open questions.

This his is joint work with M. Pouzet, N. Sauer, and R. Woodrow.

JUSTIN MOORE, Cornell University
Transcending ω_1-sequences of reals

I will describe a procedure which, to each ω_1-sequence of reals, assigns a sequence of tree orderings on ω_1 which attempts to build a real number not in the range of the sequence. If this attempt fails, the result is a tree of countable closed subsets of ω_1 which has no uncountable branch, is completely proper as a forcing notion (and remains so in any outer model with the same reals in which T does not have an uncountable branch), and is "self specializing" in the sense that

$$\{(s,t) \in T^2 : (\mathrm{ht}(s) = \mathrm{ht}(t)) \wedge (s \neq t)\}$$

can be decomposed into countably many antichains. In particular, this tree can have at most one branch in any outer model. This in particular shows that the forcing axiom for completely proper forcings is inconsistent with the Continuum Hypothesis, thus answering a longstanding problem of Shelah.

ITAY NEEMAN, UCLA
The tree property up to $\aleph_{\omega+1}$

The tree property is a combinatorial principle that resembles large cardinal reflection properties, but may hold at successor cardinals. We present some recent work showing, from ω supercompact cardinals, that the tree property can be forced to hold at all successor cardinals in the interval $[\aleph_2, \aleph_{\omega+1}]$. This is a further step in a general project of obtaining the tree property on increasingly large intervals of successor cardinals, and builds on work of Cummings–Foreman below $\aleph_\omega$, and work of Magidor–Shelah and Sinapova at $\aleph_{\omega+1}$.

ASSAF RINOT, Fields Institute and University of Toronto Mississauga
The extent of the failure of Ramsey's theorem at uncountable cardinals

We study the principle c_κ, which asserts the existence of a coloring $c : [\kappa]^2 \to \kappa$ with the property that $c[A \times B] = \kappa$ for every cofinal subsets A, B of κ.

It is established that c_κ holds whenever (1) κ is the successor of a regular cardinal, (2) κ is the successor of a singular cardinal and $\kappa \nrightarrow [\kappa]^2_\kappa$. This is joint work with Stevo Todorcevic.

GRIGOR SARGSYAN, Rutgers University
Square in Pmax extensions

We will prove the consistency of the simultaneous failure of $\square_{\omega_2}$ and $\square(\omega_2)$ in a Pmax style extension. This considerably reduces the known large cardinal strength of the axiom needed to establish the above consistency.

DIMA SINAPOVA, UC Irvine

A hybrid Prikry type forcing

Extender based Prikry type forcing is one of the most direct ways of violating the Singular Cardinal Hypothesis (SCH). We will describe a construction combining extender based Prikry and diagonal supercompact Prikry forcing. We will discuss its implications on singular cardinal arithmetic, and more precisely the relationship between SCH and combinatorial principles like Jensen's square and scales.

SLAWOMIR SOLECKI, University of Illinois

A proof of uniqueness of the Gurarii space

A Gurarii space, constructed in 1965, is a separable Banach space that is universal for separable Banach spaces and is approximately homogeneous. Uniqueness up to isometry of the Gurarii space was proved by Lusky in 1976 using deep techniques developed by Lazar and Lindenstrauss. Subsequently, another proof of uniqueness was given by Henson using model theoretic methods of continuous logic. The question whether there is an elementary proof of uniqueness occurred to several mathematicians. This question was made current by recent increased interest in universal, homogeneous structures. In the talk, I will provide just such an elementary proof of isometric uniqueness of the Gurarii space.

This is a joint work with Wieslaw Kubis.

JURIS STEPRANS, York

Michael space in the Laver model

This talk will discuss the status of Michael spaces in the Laver model.

ASGER TORNQUIST, University of Copenhagen

Conjugacy of measure preserving actions of F_n

It was recently shown by Foreman, Rudolph and Weiss that the conjugacy relation for ergodic probability measure preserving Z-actions is complete analytic. In this talk I will discuss some recent work with Inessa Epstein, where we show this also holds for ergodic p.m.p. actions of the non-amenable free groups F_n, $(n > 1)$, and many other groups. We also obtain that orbit equivalence and von Neumann equivalence of such actions are analytic non-Borel relations.

MARTIN ZEMAN, University of California at Irvine

Universality of local core models

Assume M is a proper class inner model; denote the core model computed in M by K^M. Under suitable anti-large cardinal hypothesis we show that if $\delta > \omega$ is a regular cardinal in V such that M correctly computes the its cardinal successor then $K^M||\delta$, the initial segment of K^M of length δ, is universal for all iterable premice in V of size strictly smaller than δ. I will also discuss several variations and consequences of this fact. Perhaps the most interesting consequence is that the existence of δ as above implies that M is Σ^1_3-correct. This is a joint work with Andres Caicedo.