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Catching antichains

The notion of *antichain catching* appeared in the Foreman-Magidor-Shelah paper on Martin's Maximum, and was used extensively in Woodin's proofs of the presaturation of various stationary tower forcings. For normal ideals $\mathcal{I}$ and $\mathcal{J}$, let us say that $\mathcal{J}$ *catches* $\mathcal{I}$ (and write $catch(\mathcal{J}, \mathcal{I})$) iff $\mathcal{J}$ has sufficiently large support, the $\mathcal{J}$-positive sets project onto the $\mathcal{I}$-positive sets in a certain canonical manner (as ideals), and whenever $G \subset (\mathcal{J}^+, \subset)$ is generic then the projection of G is generic for $(\mathcal{I}^+, \subset)$. Certain instances of $catch(\mathcal{J}, \mathcal{I})$ are equivalent to saturation of $\mathcal{I}$ (namely when $\mathcal{J}$ is the *conditional club filter relative to* $\mathcal{I}$; see Foreman's chapter in Handbook of Set Theory). But in general the statement:

"there exists a $\mathcal{J}$ such that $catch(\mathcal{J}, \mathcal{I})$"

is strictly weaker than saturation of $\mathcal{I}$ and strictly stronger than precipitousness of $\mathcal{I}$. I will discuss this result and others from some joint work with Martin Zeman; I will also discuss some joint work with Matteo Viale which made use of related notions.