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Empty Regions of Random Pattern-Avoiding Permutations

A *pattern* of length k is simply a permutation of $\{1, \dots, k\}$. A permutation of $\{1, \dots, N\}$ (for $N > k$) is said to avoid a specific pattern P if the (long) permutation has no subsequence of k elements that appears in the same relative order as P . (E.g. the permutation (6425713) does not avoid the pattern (132) because the permutation contains the subsequence (273).) For a given pattern P , let $S_N[P]$ be the subset of permutations of $\{1, \dots, N\}$ that avoid P . The cardinality of $S_N[P]$ has been extensively studied by combinatorialists.

This talk examines properties of random elements of $S_N[P]$. Monte Carlo experiments reveal some striking features when these random permutations are graphed as functions from $\{1, \dots, N\}$ to $\{1, \dots, N\}$. We prove that for some patterns, certain regions of $[1, N]^2$ are exponentially unlikely to contain any points of such a graph. We characterize which patterns produce such “empty regions.”

This is joint work with Mahshid Atapour.