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Scalar Curvature for the Noncommutative Two Torus I

We give a local expression for the *scalar curvature* of the noncommutative two torus $A_\theta = C(\mathbb{T}^2_\theta)$ equipped with an arbitrary translation invariant complex structure and Weyl conformal factor. The metric information is encoded in the Dirac operator D of a *twisted spectral triple* on A_θ so that we view this C^*-algebra as a noncommutative Riemannian manifold. The local expression for curvature is computed by evaluating the value of the (analytic continuation of the) *spectral zeta function*

$$\zeta_a(s) := \mathsf{Trace}(a|D|^{-s})$$

at $s = 0$ as a linear functional in $a \in C^\infty(\mathbb{T}^2_\theta)$. A new, purely noncommutative, feature here is the appearance of the *modular automorphism group* from the theory of type III factors and quantum statistical mechanics in the final formula for the curvature. This formula coincides with the formula that was recently obtained independently by Connes and Moscovici in their recent paper. At the end, we will explain how this formula fits into our earlier work on *Gauss-Bonnet theorem* for noncommutative two tori, which extends the Gauss-Bonnet theorem of Connes and Tretkoff to general conformal structures on $\mathbb{T}^2_\theta$. This is joint work with Farzad Fathizadeh.