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**Mathematics Education**  
**Éducation mathématique**  
(Org: **Walter Whiteley** (York))

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**OLIVE CHAPMAN**, University of Calgary

*Supporting Mathematical Thinking and Processes in the Mathematics Curriculum*

Mathematical thinking and mathematical processes are central to mathematics education as ways of doing and learning mathematics with meaning and for the development of contextual understanding. The National Council of Teachers of Mathematics [NCTM] strongly promotes five mathematical processes in learning mathematics. More recently NCTM has emphasized the importance of sense making and reasoning in high school mathematics noting that these are the foundations of the five processes. These processes often take a back seat in traditional mathematics classrooms where the focus tends to be on procedural approaches to do and learn mathematics. So, what is mathematical thinking? What are these mathematical processes? Why are they important? What are ways of supporting students' development of mathematical thinking and engagement in mathematical processes in doing and learning mathematics? I focus on these questions in this presentation. I discuss a cognitive view of mathematical thinking, the mathematical processes advocated by NCTM, the relationship between mathematical thinking and the mathematical processes, and how these processes are considered in the school mathematics curriculum (using Alberta and Ontario Programs of Studies as examples). Based on the research literature in mathematics education, I also highlight examples of categories of mathematical tasks that support mathematical thinking and processes. Based on my teaching of mathematics education courses for prospective secondary mathematics teachers and my research on exemplary practicing secondary mathematics teachers' thinking and teaching of algebra and problem solving, I provide examples of inquiry-based pedagogical approaches to support the development of mathematical thinking and learning of the processes.

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**SHAWN GODIN**, Cairine Wilson S.S.

*Promoting Thinking Through the Mathematical Processes*

In their 2005 revision of the K - 12 mathematics curriculum in Ontario, the Ministry of Education introduced seven *mathematical processes* that teachers are to keep in mind while teaching and evaluating students. The processes are really, for the most part, higher order thinking skills that are valuable for **all** students.

In this talk, I will outline my attempts to highlight the processes with my students, particularly the process expectation *reflection*. Examples of evaluation items and student responses will be shared.

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**ANN KAJANDER**, Lakehead University

*Big Ideas in Bite-Size Pieces: Teacher Knowledge of Mathematics*

A growing body of research argues that elementary teachers' ability to support learning is related to their understanding of a specialised domain of mathematics, variously referred to as pedagogical content knowledge or mathematics for teaching. For our purposes, we refer to this knowledge as mathematics for teaching and learning (MTL). Based on our six year study of 573 upper elementary preservice teachers (as well as about 100 in-service teachers), we argue that preservice teachers' knowledge of elementary mathematics remains highly procedural and rule-based, especially upon entry to the education program. Our evidence further suggests that the perception of mathematics as a procedural rule-based subject also persists among many in-service teachers. According to our database related to MTL, preservice teachers who experienced learning in the earlier (i.e.1997) version of the current Ontario elementary curriculum since even as early as grade 4, could not be argued as conceptually stronger than those who were in school prior to this curriculum revision. Our data suggest that preservice teachers still typically perceive mathematics as "something to memorize", with explanations taken to mean simply stating a rule. Teacher candidates with strong mathematics backgrounds were initially only marginally stronger conceptually than their peers. Recent mathematical interventions at our institution include an optional course in 'mathematics for teaching' as well as a mandatory high stakes examination in MTL. These changes have appeared to support stronger participant growth during

the teacher education program, but results remain less than satisfactory. Implications for curriculum development, given this climate, will be discussed.

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**MARGO KONDRATIEVA**, Memorial University  
*Case-invariant proofs in a dynamic geometry environment*

When doing *proof by cases* on paper very often we need to come up with different ideas and techniques in each particular case of a more general situation, and draw each particular case separately. However, in a dynamic geometry environment (DGE) a smooth visual transition between different cases is often available. For example, one may easily pass from the case of obtuse triangle to the case of acute triangle by *dragging* a vertex of this triangle. In this presentation we are interested in constructions of geometrical solutions that are valid in all possible cases of a given problem and their case-invariance is observable by *dragging* base points of a dynamic drawing (applet). We discuss examples of problems from Euclidean geometry and their case-invariant solutions produced in a DGE.

In each of our examples the discussion of a case-invariant solution has a slightly different emphasis. In the first example we demonstrate the importance of consideration of special cases: the key contraction that was found in a special case of our problem suggested the solution to the original problem taken in full generality. In the second example we illustrate the possibility to notice some additional geometrical facts useful for proving other statements while looking at different cases of a theorem's proof. Another example shows that trying to unify various cases of a problem using a DGE may allow one to deepen their understanding of certain geometrical notions (such as area) and make connections with other branches of mathematical knowledge.

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**KATHY KUBOTA & PAT MARGERM**, Toronto Catholic District School Board and York University  
*The Complexities of Teachers Using Big Ideas in Mathematics*

This session aims at developing perspectives on the challenges that pre-service and in-service teachers face in understanding and using big ideas in mathematics for preparing and carrying out lessons in the classroom. To address these challenges, aspects of teacher's daily mathematical work are analysed in order to conceive possible strategies for developing teacher's understanding and use of big ideas in mathematics for their teaching.

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**MIROSLAV LOVRIC**, McMaster University  
*Learning Mathematics in Interdisciplinary Context*

The aim of my research is to investigate the impact of an interdisciplinary program (in particular, McMaster's iSci = Interdisciplinary Science Program) on learning of mathematics in the first year of university. To what extent does the rich interdisciplinary learning environment enhance and deepen learning, both in terms of content knowledge and mathematical skills (formation of a precise mathematical argument, communication of scientific ideas, etc.)?

A pre-test/ post-test scheme is used to collect the evidence. In the first week of classes in September, students are administered an unannounced 50-minute survey, which gives an initial assessment of their general math knowledge and skills. After students complete the survey, no aspect of it is addressed in lectures. Eight months later, at the end of the school year, students are given the same survey, again unannounced. What is the purpose? None of the survey questions are explicitly discussed in lectures. However, throughout the first-year instruction in iSci students are exposed to a number of activities (such as problem-solving, critical thinking, creating precise scientific arguments, and so on), which can help them answer test questions better than at the start. The purpose of this approach is to determine whether students did learn math in the sense of being able to apply it to situations that were not explicitly addressed in lectures.

In my talk I will present the data, discuss interesting and relevant findings, and comment on the implications for other disciplines, within as well as beyond the iSci program.

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**PANEL ON STATISTICAL INFERENCE**, University of Toronto  
*Big Ideas in Statistical Reasoning*

H.G. Wells is often quoted as saying that "Statistical thinking will one day be as necessary a qualification for efficient citizenship as the ability to read and write." Yet we're also told that there are three types of lies: "lies, damned lies and statistics". As citizens, we are bombarded with data representations, measures of central tendency, and probabilistic statements through websites, blogs, print media, and broadcast media. Students will encounter statistics and probability in virtually every avenue of their academic careers, be it in the social sciences, the humanities, or the natural sciences.

Like mathematical reasoning, statistical reasoning requires clear, logical thought. But in contrast to the certainty prevalent in mathematical arguments, statistical reasoning requires the use of rational thought to make sense of uncertainty. We will explore issues relating to integrating stochastic thinking—both process and content—into the mathematics curriculum across all grades, from K–14. Questions might be based on the relative importance of intuitive, experimental, and theoretical probability; the difference between causality and correlation; understanding randomness; or teacher preparedness in terms of understanding and embracing uncertainty and how to best teach the concepts and processes.

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**MEDHAT RAHIM**, Lakehead University

*Reasoning, Conjecture Making and Spatial Structuring by High School Student-Teachers and the Radical Constructivist Paradigm*

In a classroom based research, a series of tangram related tasks, focusing at reasoning, sense making and conjecturing were utilized. Sociocultural and psychological components of von Glasersfeld Theory of radical constructivism and Battista concept of spatial structuring have been the basis for the researchers' observations and tasks' analyses. The main purpose for this research was to describe and analyze high school teacher candidates' initial cognitive constructions, their modification, and re-modification of their responses as they were proceeding in their attempts to justify their responses.

The 'tangram' has been originally referred to as the 7-pieces dissection (or tangram problem) consisting of seven flat shapes forming together a square shape (five triangles: two identical large, two identical small and one medium triangles; a small square and a parallelogram). It was originally invented in China at some unknown year in history, and then carried over to the world by trading ships in the early 19th century (Wang and Hsiung, 1942; Read, 1965). In particular, assuming that the small square has an area of one unit square then each large triangle has an area of two unit square and each small triangle of an area of half unit square and medium triangle of an area of one unit square and a parallelogram have an area of one unit square too. In collaboration with Radcliffe Siddo, Lakehead University (rsiddo@lakeheadu.ca)

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**TINA RAPKE**, University of Calgary

*Incorporating the practices of mathematicians into the classroom*

This talk will examine how and why one might incorporate the practices of mathematicians into the classroom. I will describe a few practices of mathematicians and how they can be used to encourage student engagement in mathematics. I will discuss the implications of employing such practices for teacher education and curriculum.

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**MARIAN SMALL**, University of New Brunswick

*How and why is it useful to use big ideas in K-12 math?*

This seminar will explore, with examples attending to both elementary and secondary curriculum, the following issues:

Although we could, from a strictly academic perspective, develop a set of big ideas to encompass K-12 mathematics, the real issue is whether and/or how this is of value in the classroom setting.

- Is it the teacher who is assisted by this focus, the students, or both?

Is the main value of focusing on big ideas to provide rich enough connections for students that new content related to those big ideas becomes more accessible? Or is the main value to streamline the curriculum? Teachers are often concerned that there is too much to teach and they see a focus on big ideas as a way to have less to teach.

- How would classroom practice be affected?

Would it really change the way in which instruction is offered to students? Is it more about the problems that are posed or the questions that are asked about those problems? Or would it simply be a change in optics?

- Should big ideas be the "content equivalent" of processes, i.e. those content ideas that are revealed repeatedly in different guises throughout the curriculum or should the big ideas be the processes themselves? Is it enough for the big ideas to be just the processes?

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**WALTER WHITELEY**, York University  
*Transformations and Invariance - a Big Idea?*

A big idea of modern geometry centers on transformations and invariance (what is not changed by the transformations). Since 1870's, the definition of "a geometry" has been a set of objects, a group of transformations, and the study of properties unchanged by the transformations (Felix Klein's Erlanger Program (i) If the transformations are translations, rotations and reflections - distance preserving congruence maps or isometries, the properties include distances and angles, and we have Euclidean Geometry. (ii) If we allow more transformations (scaling, differential scaling in one coordinate) we get affine transformations (what the sun light does to a picture on the window to the shadow on the floor), and affine geometry. What is preserved? What is now changed (distances in different directions) and what is unchanged (parallel lines are still parallel)

In physics, picking your coordinate frame and starting time do not change the solution of the problem. The laws of physics are unchanged by where the origin is, what the 'start time is' or what direction you call 'x'. Choosing your frame of reference is a transformation of the equations for the laws of physics, and the answer transforms in the same way - it is invariant. In physics, this 'symmetry in the laws' is recognized as a big idea and connects to conservation laws, through Noether's Theorem.

How do these connect to how children learn - and how the curriculum can be structured? Are there other examples back in arithmetic, in algebra, in statistics ... ?

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**CHRIS WILD**, University of Auckland  
*The new Mathematics and Statistics Curriculum of New Zealand*

We will discuss the general principles about big ideas and competencies that pervade all of the discipline-specific New Zealand curricula and how this has played out in mathematics and statistics. We will then dig down into the statistical strands. Our new statistics curriculum is the most modern and ambitious anywhere internationally and is notable for the development of broad-process rather than detail oriented strands through time. We will discuss how this came to be and the processes whereby the curriculum ended up largely being written by consensus of a fairly large discussion-group representing the statistical profession, academic statisticians, teacher educators and developers, and leading teachers from across the country. We will move on to consider the downstream issues, and dangers, in terms of teacher development and student assessment. Differences between how we were thinking about things in NZ and how you are thinking in Canada will, I hope, be a useful way of triggering some useful discussions.