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Transformations and Invariance - a Big Idea?

A big idea of modern geometry centers on transformations and invariance (what is not changed by the transformations). Since 1870's, the definition of "a geometry" has been a set of objects, a group of transformations, and the study of properties unchanged by the transformations (Felix Klein's Erlanger Program (i) If the transformations are translations, rotations and reflections - distance preserving congruence maps or isometries, the properties include distances and angles, and we have Euclidean Geometry. (ii) If we allow more transformations (scaling, differential scaling in one coordinate) we get affine transformations (what the sun light does to a picture on the window to the shadow on the floor), and affine geometry. What is preserved? What is now changed (distances in different directions) and what is unchanged (parallel lines are still parallel)

In physics, picking your coordinate frame and starting time do not change the solution of the problem. The laws of physics are unchanged by where the origin is, what the 'start time is' or what direction you call 'x'. Choosing your frame of reference is a transformation of the equations for the laws of physics, and the answer transforms in the same way - it is invariant. In physics, this 'symmetry in the laws' is recognized as a big idea and connects to conservation laws, through Noether's Theorem.

How do these connect to how children learn - and how the curriculum can be structured? Are there other examples back in arithmetic, in algebra, in statistics ... ?