History and Philosophy of Mathematics
Histoire et philosophie des mathématiques
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TOM ARCHIBALD, Simon Fraser University
Galois and the Galois Theory of Differential Equations

It took decades before the French mathematician Évariste Galois became the national and international mathematical icon that is now familiar. However, we shall show that this was well-established by the 1880s, and that competition to become the true heir to Galois' thought was fierce in the context of establishing a "Galois theory" of differential equations. At the time, Picard, Vessiot, Drach and others display competing views of the essential character of Galois' achievement. In this paper we assess these contrasting views, comparing them to the image of the mid-19th century and to that today.

DAVID BELLHOUSE, University of Western Ontario
Mathematicians in the British Financial Revolution of the Eighteenth Century

Some modern historians of science have argued that during the eighteenth century practicing mathematicians had little or no impact on the pricing of life contingent products that were sold in the marketplace, although the same mathematicians wrote several books on the fair pricing of these products. For the first two-thirds of the eighteenth century, the major life contingent product was the life annuity. Annuities were sold both on the open market for privately financed schemes and through the government to finance the national debt. Through the use of several examples, it is shown that that there was a diffuse, but perhaps not very large, group of mathematicians active in the annuity and insurance marketplace during the eighteenth century. These mathematicians acted as consultants to those buying or selling annuities in the marketplace. It is argued that publication of their books on annuities was, in part, a form of self-advertisement and an establishment of the credibility of their qualifications.

WILLIAM HACKBORN, University of Alberta
Tartaglia and Galileo: On Motion and Ballistics

This talk will examine contributions of two mathematicians, Niccolò Tartaglia (1500-1557) and Galileo Galilei (1564-1642), to the science of motion and the theory of exterior ballistics. Tartaglia, best known for discovering how to solve cubic equations, wrote the first text on exterior ballistics, *Nova Scientia* (1537), in which he applied Euclidean reasoning and Aristotle's ideas on motion to the trajectories of cannonballs. In his own study of motion, Galileo also wrestled with Aristotelian ideas, first in his manuscript *De Motu* (1591) and much later, after a kind of personal paradigm shift, in *Two New Sciences* (1638), in which he laid the mathematical foundation for much of Newtonian mechanics and applied his new science to ballistics. In addition to looking closely at key propositions of Tartaglia and Galileo, this talk will also briefly consider their work in the broader context of applied mathematics.

DEBORAH KENT, Hillsdale College
Networks and linkages in nineteenth-century mathematics journals

CaaFÉ (Circulation des saviors et pratiques algébriques et arithmétiques (1870-1945): Sources et échanges: France, Europe, États-Unis) is a collaborative research project seeking to understand how knowledge of algebra and number theory circulated between Europe and the United States from 1870 to 1945. Our team is using the online collective database THAMOUS to create links between mathematical articles and monographs of the period. The project clarifies paths of circulation and linkages based on networks of references that mathematicians made at the time of their research. This talk will demonstrate the methodology using examples from late nineteenth-century American journals.

ALEX KOO, University of Toronto
Examining the Relationship Between Internal and External Mathematical Explanations

Mathematical explanations come in two general varieties: mathematical explanations of mathematical facts and mathematical explanations of physical facts, henceforth called internal and external explanations respectively. The debate in internal explanations boils down to identifying and understanding the difference between a demonstration of a mathematical fact, and an explanation of said fact. Meanwhile, the debate within external explanations attempts to determine whether or not mathematics can genuinely explain physical, non-mathematical phenomena.

One important question that has received little attention is how internal and external mathematical explanations relate to each other, if at all. It seems clear that if one type of explanation is related to the other it would be external explanations that would depend on internal explanations. Mark Steiner (1978) was the first to propose such a relationship. Steiner claimed that an external explanation is such that if you 'remove the physics' from the explanation, what remains is an internal explanation of a mathematical truth. The suggestion here is that external explanations owe some of their explanatory power to the existence of a good internal explanation.

In this paper I will survey several different theories on internal explanation and examine whether or not Steiner's suggestion of dependence is tenable. Recent accounts of external explanation seem to assume that there is an intimate relationship between internal and external explanation. I aim to show that such an assumption is unjustified as not all accounts of internal explanation are suitable for such a dependence.

JEMMA LORENAT, Simon Fraser University
Not set in stone: nineteenth century geometrical constructions and the Malfatti Problem

In 1803, Gian Francesco Malfatti posed the problem of constructing three parallel cylinders of maximal volume from a marble triangular prism. He then reduced the problem to that of inscribing three mutually tangent circles in a triangle. This apparently equivalent construction reappeared repeatedly through the nineteenth century. A decade after Malfatti, Joseph-Diaz Gergonne posed the problem to the readers in the first issue of his Annales. In the same volume, he gave one of the three analytic solutions, admitting that he had struggled for several years. The construction gained a wider audience when Jakob Steiner offered a purely synthetic treatment in his 1826 article published in the first volume of Crelle's Journal. Many, including Gergonne, found Steiner's proof incomplete, and alternative approaches continued to emerge in France, Germany, and later Britain and the United States. The so-called Malfatti problem gained notoriety in recent decades when the accepted reduction was shown to be false – in fact, three mutually tangent circles never provide the optimal solution (Zalgaller and Los, 1992). Nevertheless, the dispersion of the Malfatti problem provides an illustrative thread through the evolution of nineteenth century geometry. From this perspective we observe efforts towards developing general theories to encompass the approaches for particular problems, the differentiation and com- petition of geometric methodologies, and the nationalization and internationalization of mathematical communities.

SYLVIA NICKERSON, IHPST, University of Toronto
Printing mathematics using moveable type and engraving

In the nineteenth century, printing mathematics was considered a specialized art within the printing trade, and not every publisher undertook it. Translating mathematical ideas into print presented technical challenges to both printers and authors. Works laden with symbols and diagrams had to be set in moveable type, a technology designed to replicate alphabetic script. Diagrams had to be engraved with precision to preserve the meaning the author intended. Looking at three examples of mathematical books published by English Victorian publishing houses, I will examine how these mathematical authors and their publishers surmounted the difficulty of printing mathematics using movable type and engraving.

BRUCE PETRIE, University of Toronto
Properly Understanding Lambert's Mémoire on pi

Johann Heinrich Lambert (1728-1777) is remembered for being the first to prove the irrationality of pi. He presented this proof in his 1768 memoire which he submitted in 1761 to the Royal Academy of Sciences in Berlin. While a crowning achievement in and of itself, Lambert's result deserves to be remembered for a different reason as well. In this work, Lambert was the first to conceive of transcendental numbers as numbers which are not roots of polynomials with rational coefficients. While his memoire is mentioned in anecdotal accounts of squaring the circle, these accounts miss this pivotal development. Squaring the circle did indeed motivate much of Lambert's research, which was only natural given his interest in geometry and perspective. The problem is mentioned frequently in his memoire and motivated his proof that pi is irrational. I show that it was also the motivation behind his revision of the vague Eulerian understanding of transcendence.

JOSIPA PETRUNIC, University of Toronto, IHPST

Platonism, Cognitive Science and Learning Mathematics: a Wittgensteinian response to exclusionary trends in philosophy of maths

The philosopher of mathematics, James Brown, has argued that mathematical objects exist in an atemporal realm. We gain knowledge of these objects through a special intuition. This unique knowledge is filtered through the conduit of an expert, a philosopher king or a really smart mathematician who can show us the way. Speaking from the perspective of Kantianism the philosopher of mathematics, Marcus Giaquinto, has claimed we necessarily come to "know" mathematical objects over time, because our brains are hardwired to grasp their special intuitive existence. Brown and Giaquinto's arguments boil down to much the same thing: mathematical objects exist outside of human life; they can be discovered by humans, but not created by humans. Both Brown and Giaquinto view mathematics as a special domain of thinking (different from political, economic, religious or other social thinking). Both authors believe mathematics requires special people to do the thinking. Neither Brown's Platonism nor Giaquinto's neo-Kantianism explains why we "know" mathematical theorems or mathematical objects as we know them today. Neither account offers an explanation as to why we use the mathematical techniques we use. Neither account explains why we pose the mathematical questions that we pose. A better explanatory account of mathematical activity and mathematical knowing comes from Wittgenstein, who would argue what we know in mathematics and what we use in mathematics is based on what we are taught, what we learn, and what the social realm of possible extension of that knowledge to new case studies permits.

AARON WRIGHT, University of Toronto and Massachusetts Institute of Technology

Penrose Diagrams and the "Renaissance" of General Relativity, 1962–1973

Penrose diagrams manifest the impossible: infinite universes, black holes, even *chains* of infinite universes *connected* by black holes become finite and definite on the page. This was accomplished through the development of ideas from topology within physics. The conceptual shifts engendered by the diagrams and the conformal transformations used to create them produced new ways of understanding mathematical- and physical-concepts. They were invented by Roger Penrose in 1962 in the context of the study of Einstein's theory of gravity, General Relativity. This was during a period of growth and development in the field that has been called the "Renaissance" of General Relativity. In this paper I trace the path of Penrose diagrams through the community of mid-century "relativists." This path highlights a structural feature of this community—the close interrelation of the contexts of research and of pedagogy—that helps explain the occurrence of this "Renaissance." The diagrams are taken to be an exemplar of the role of new "paper tools" in physics. Penrose diagrams are part of a visual tradition of space-time diagrams within physics; they are also part of Penrose's personal tradition of exploring the art and psychology of M. C. Escher-like "impossible objects." My analysis highlights the convergence of new ways of thinking and new ways of seeing in General Relativity's "Renaissance."