Financial Mathematics
Mathématiques financiers
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ALEXANDER ALVAREZ, Ryerson University
Local continuity of stopping times and arbitrage

In this work we extend some of the results in [Bender, Sotinnen and Valkeila(08)] to prove the absence of arbitrage in markets driven by non semimartingale models. To this end, we restric the portfolio strategies to those that depend on locally continuous stopping times relative to a metric structure in the trajectory space. Technically we rely on a non-probabilistic Ito's formula for functions with finite quadratic variation. We discuss some implications of our results and prove absence of arbitrage for non semimartingale models having jumps or stochastic volatility. For the analized examples we prove the corresponding small balls properties and the local continuity of the porfolio value under different metrics.

ALEX BADESCU, University of Calgary
Hedging GARCH options with generalized innovations

In this paper, we study the performance of different hedging schemes when the asset return process is modelled by a general class of GARCH models. Since the minimal martingale measure fails to produce a probability measure in this setting, we construct local risk minimization (lrm) hedging strategies with respect to a risk neutral measure. Using the conditional Esscher transform and the extended Girsanov principle as our martingale measure candidates, we construct lrm delta hedges based on different distributional assumptions regarding the GARCH innovations. An extensive numerical experiment is conducted to compare these hedges to the standard stochastic volatility delta hedges for different European style option maturities and hedging frequencies.

GERMAN BERNHART, TU München
Numerical density calculation for distributions of the Bondesson class

We address the numerical density calculation via Laplace inversion for distributions of the Bondesson class. The classical Bromwich inversion integral involves serious computational challenges such as highly oscillating integrands and infinite integration bounds. It is proven that a certain contour transformation is admissible for the considered class of distributions, yielding a rapidly declining integrand and allowing for a substitution to a finite interval. The approach is tested for distributions with known density (Gamma distribution, IG distribution) and compared to other approaches for unknown densities (alpha-stable distribution). Analogous procedures can be applied for the efficient numerical pricing of CDO contracts in specific CIID models. The talk is based on the paper "Numerical density calculation for distributions of the Bondesson class" by G. Bernhart, J.-F. Mai, S. Schenk, and M. Scherer.

JOE CAMPOLIETI, Wilfrid Laurier University
Dual Stochastic Transformations of Solvable Diffusions

We present new extensions to a method for constructing several families of solvable one-dimensional time-homogeneous diffusions. Our approach is based on a dual application of the so-called diffusion canonical transformation method that combines smooth monotonic mappings and measure changes via Doob-h transforms. This gives rise to new multi-parameter solvable diffusions that are generally divided into two main classes; the first is specified by having affine (linear) drift with various resulting nonlinear diffusion coefficient functions, while the second class allows for several specifications of a (generally nonlinear) diffusion coefficient with resulting nonlinear drift function. The theory is applicable to diffusions with either singular and/or non-singular endpoints. As part of the results in this paper, we also present a complete boundary classification

and martingale characterization of the newly developed diffusion families. The first class of models, having linear drift and nonlinear (state-dependent) volatility functions, is useful for equity derivative pricing in finance, while the second class of diffusions contains new models that are mean-reverting and which are applicable in areas such as interest rate modeling. As specific examples of the first class of affine drift models, we present explicit results for three new families of models. For the second class of nonlinear drift models, we give examples of solvable subfamilies of mean-reverting diffusions and derive some closed-form integral formulas for conditional expectations of functionals that can be used to price bonds and bond options.

BARBARA GOETZ, TU München

Valuation of multi-dimensional derivatives in a stochastic correlation framework

Stochastic volatility models have been in place for some years now. A natural extension of the latter ones is a multivariate model with stochastic correlation. And indeed, the performance of a portfolio or a multi-dimensional derivative depends very much on the joint behaviour of the underlyings, i.e. the covariances, which are not constant over time. However, one of the main problems with the modelling of correlation is intractability because the number of parameters grows quite fast as dimensions increase. The model treated here is based on a stochastic principal component model, which reduces the dimension of the original problem. We reduce complexity by modelling the eigenvalues of the assets instead of the full covariance matrix. We set the eigenvectors constant but assume the eigenvalues stochastic. An empirical analysis shows that the eigenvalues are driven by two mean-reverting components, one which varies in the order of days and the other one which varies in the order of months. Our approach allows a multi-dimensional extension of the Heston model with stochastic volatility, stochastic correlation among assets, between variances and assets as well as between assets and correlation. The proposed model is applied to price end-point as well as path-dependent two-asset options. A closed-form solution for barrier options under stochastic correlation has not been found. Thus, we show how perturbation theory can be used to find easy and well converging approximations to non-vanilla options on two correlated underlyings.

MATHEUS GRASSELLI, McMaster University

An agent-based computational model for bank formation and interbank networks

We introduce a simple framework where banks emerge as a response to a natural need in a society of individuals with heterogeneous liquidity preferences. We examine bank failures and the conditions for an interbank market is to be established.

We start with an economy consisting of a group of individuals arranged in a 2-dimensional cellular automaton and two types of assets available for investment. Because of uncertainty, individuals might change their investing preferences and accordingly seek their surroundings neighbours as trading partners to satisfy their new preferences. We demonstrate that the individual uncertainty regarding preference shocks coupled with the possibility of not finding a suitable trading partners when needed give rise to banks as liquidity providers. Using a simple learning process, individuals decide whether or not to join the banks, and through a feedback mechanism we illustrate how banks get established in the society. We then show how the same uncertainty in individual investing preferences that gave rise to banks also causes bank failures. In the second level of our analysis, in a similar fashion, banks are treated as agents and use their own learning process to avoid failures and create an interbank market.

In addition to providing a bottom up model for the formation of banks and interbank markets, our model allows us to address under what conditions bank oligopolies and frequent banks failures are to be observed, and when an interbank market leads to a more stable system with fewer failures and less concentrated market players.

TOM HURD, McMaster University

Analyzing contagion in banking networks

I introduce a class of stylized banking networks and try to predict the size and frequency of contagion events. I find that the domino effect can be understood as an explicit iterated mapping on a set of edge probabilities that converges to a fixed point. A cascade condition is derived that characterizes whether or not an infinitesimal shock to the network can grow to a finite size cascade, in analogy to the basic reproduction number R_0 in epidemic modeling. It provides an easily computed measure of the systemic risk inherent in a given banking network topology. An analytic formula is given for the frequency of global cascades,

derived from percolation theory on the random network. Two simple examples illustrate that edge assortativity can have a strong effect on the level of systemic risk as measured by the cascade condition. Although the analytical methods are derived for infinite networks, large-scale Monte Carlo simulations demonstrate the applicability of the results to finite-sized networks. Finally, we'll see that a simple graph theoretic quantity, graph assortativity, seems to best capture systemic risk.

CODY HYNDMAN, Concordia University

Generalized filter-based EM algorithm and applications to calibration

The Kalman filter has been applied to a wide variety of financial models where the underlying stochastic processes driving a price are unobservable directly. Maximum likelihood parameter estimation for these models is challenging due to the recursive nature of the Kalman filter as well as the complicated interdependence of the signal and observation equations on multiple parameters. An alternative to direct numerical maximization of the likelihood function is the Expectation Maximization (EM) algorithm producing a sequence of parameter estimates involving two steps at each iteration: the Expectation step (E-step) and Maximization step (M-step). The filter-based approach developed in Elliott and Hyndman [J. Econom. Dynam. Control 31 (2007), no. 7, 2350–2373] requires only a forward pass through the data and is therefore potentially twice as fast as the smoother-based algorithm. The filter-based algorithm is expressed in terms of decoupled filters that can be computed independently in parallel on a multiprocessor system allowing for further gains in efficiency. In this paper we derive new finite-dimensional filters which allow the EM algorithm to be implemented for certain multi-factor commodity price models, generalizing the results of Elliott and Hyndman [op. cit.]. In the cases under consideration the solution to the M-step does not exist in closed form. However, it is possible to approximately solve the M-step by applying one-iteration of Newton's method to the high degree polynomials characterizing some of the updated parameters resulting in a Generalized EM algorithm. The method is illustrated by application to a two-dimensional commodity price model.

SEBASTIAN JAIMUNGAL, University of Toronto

Buy Low, Sell High: A High Frequency Trading Perspective

In this I will present a class of self-exciting processes as a promising approach to modeling trading activity at high frequencies. Our model neatly accounts for the clustering of intensity of trades and the feedback effect which trading induces on both market orders as well as the shape of the limit order book (LOB). Further, it allows for efficient calibration to market data based on pseudo-likelihood methods. As well, various probabilistic quantities of interest such as the probability that the next market order is a buy or sell, the distribution of the time of arrival of a buy or sell order, and the probability that the mid-price moves a given amount before a market order arrives are also easily computable. Finally, we study an optimal control problem for a trader who places immediate-or-cancel limit buy-and-sell orders to take advantage of the bid-ask spread. Asymptotic expansions in the level of risk-aversion lead to closed form and intuitive results which are also adapted to the state of the market. Some numerical experiments will be used to demonstrate the utility of the model and optimal strategies.

[This is joint work with Alvaro Cartea, U. Carlos III de Madrid and Jason Ricci, U. Toronto]

R KULPERGER, University of Western Ontario

Multivariate GARCH

Multivariate GARCH models are an interesting time series model in finance. We discuss issues about the estimation, consistency and asymptotic normality of the estimators. These models also have many parameters, so some form of parameter reduction is needed. LASSO is a method that is useful for parameter reduction in linear regression. While this work is preliminary we are studying the use of LASSO type ideas in time series and multivariate GARCH.

ALEXEY KUZNETSOV, York University

Cool Math behind Asian options

Since Asian options were first introduced in Tokyo in 1987, there have appeared almost 3600 research papers related to these financial derivatives (according to Google Scholar web search). One may wonder, what makes these particular options

so attractive to researchers in Mathematical Finance? We think that one of the reasons is that there is a lot of beautiful Mathematics related to pricing Asian options. The goal of this talk is to discuss some of the mathematical theories involved in pricing Asian options, both in the classical Black-Scholes setting and in the more general case of Levy driven models. In particular, we will discuss the connections with self-similar Markov processes and the Lamperti transformation, the recent results of N.Cai and S.Kou on Asian options for processes with hyper-exponential jumps, and our recent results on processes with jumps of rational transform.

ROMAN MAKAROV, Wilfrid Laurier University
Pricing occupation-time derivatives

New simulation algorithms and analytical methods for pricing occupation-time derivatives under jump-diffusion processes and solvable nonlinear diffusion models are developed. A new efficient method for exact sampling from the distribution function of occupation times of a Brownian bridge is proposed. The method is applied to the exact pricing of continuously-monitored occupation-time derivatives under the double-exponential jump-diffusion process. In Monte Carlo methods for nonlinear solvable diffusion models, the occupation time is estimated using the Brownian bridge interpolation. In the second part of this talk, we consider a special family of occupation-time derivatives namely proportional step options introduced by Linetsky in [Math. Finance, 9, pp. 55-96 (1999)]. We develop new spectral expansion methods for pricing such options. Our approach is based on the application of the Feynman-Kac formula and the residue theorem. As an underlying asset price process we consider a solvable nonlinear diffusion model such as the constant elasticity of variance (CEV) diffusion model and state-dependent-volatility confluent hypergeometric diffusion processes.

This is joint work with Joe Campolieti and Karl Wouterloot.

ROGEMAR MAMON, University of Western Ontario
Weak HMM and its application to asset price modelling

A higher-order hidden Markov model (HMM) is considered in modelling the price dynamics of a risky asset. The log returns of asset prices are governed by a higher-order or the so-called weak Markov chain in a finite-state space. The optimal estimates of the second-order Markov chain and model parameters are derived. This is done via a transformation that converts the second-order HMM into the usual HMM. The model is implemented to a dataset of financial time series and its forecasting performance investigated. An extension of the parameter estimation framework is developed to handle multivariate time series data. The use of higher-order HMM captures both the regime-switching behaviour and long-range dependence in the financial data. (This is a joint work with X. Xi, Dept of Applied Mathematics, University of Western Ontario.)

ADAM METZLER, University of Western Ontario
Approximating American Option Prices via Sub-Optimal Exercize Strategies

In this talk we investigate the approximate pricing of American put options by optimizing over sub-optimal excercize strategies. Strategies are taken to be hitting times of the stock price (geoemtric Brownian motion) to smooth curves, and all curves considered are drawn from parametric families which admit closed-form first-passage time distributions. This allows one to express option values as (very well-behaved) one-dimensional integrals which are easily evaluated numerically. Despite the apparent simplicity of the method it appears to be remarkably accurate, providing an extremely rapid lower bound on the option value. The talk is based on the M.Sc. thesis of W. Xing.

DAVE SAUNDERS, University of Waterloo
Mathematics of Credit Risk Capital in the Trading Book

As part of the regulatory response to the global financial crisis, the Basel Committee on Banking Supervision has revised its rules for determining regulatory capital for credit risk in a bank's trading book. I will discuss mathematical problems related to the calculation of capital under the new regulations.

ANATOLIY SWISHCHUK, University of Calgary
Variance Swap for Local Lévy based Stochastic Volatility with Delay

The valuation of the variance swaps for local Levy based stochastic volatility with delay (LLBSVD) is discussed in this talk. We provide some analytical closed forms for the expectation of the realized variance for the LLBSVD. As applications of our analytical solutions, we fit our model to 10 years of S&P500 data (2000-01-01–2009-12-31) with variance gamma model and apply the obtained analytical solutions to price the variance swap. This is a joint talk with K. Malenfant.

TONY WARE, University of Calgary
Splitting methods in computational finance

Operator splitting methods form a staple part of our arsenal of approaches to the numerical solution of PDEs. They work by a 'divide and conquer' approach, reducing a complex problem to a sequence of simpler problems, which confers advantages when it comes to designing, coding and analyzing algorithms. We discuss some uses of operator splitting methods for certain types of Hamilton-Jacobi-Bellman equations arising in finance. We will also illustrate how operator splitting can be used to extend the applicability of existing methods to more complex settings; for example, we show how Fourier methods can be applied to option valuation problems with non-constant coefficients or in high dimensional settings.