Delay Differential Equations
Équations différentielles à retard
(Org: **Sue Ann Campbell** (Waterloo) and/et **Xingfu Zou** (Western))

JACQUES BELAIR, Université de Montréal
Modelling myelosuppression with a stage-structured model

The destruction of neutrophils (white blood cells) is a common side effect of chemotherapy. We present a stage-structured model of neutrophil regulation taking the form of a system of nonlinear delay-differential equations. The stability of its equilibria are determined, and possible Hopf bifurcations are detected. We then add the kinetic of a chemotherapeutic drug to this system, and analyze the ensuing periodic system as a function of the drug parameters to determine the amplitude and frequency of the solutions.

PIETRO-LUCIANO BUONO, University of Ontario Institute of Technology
Symmetric delay-coupled rings of Lang-Kobayashi laser models

I will present recent results on Compound Laser Mode (CLM) solutions and their bifurcations in delay-coupled rings of Lang-Kobayashi equations with cyclic and dihedral symmetry groups. Group-theoretic methods are used to classify CLMs and their bifurcations. Numerical computations with Maple and DDE-Biftool are used to identify steady-state and Hopf bifurcation points on the CLMs. The numerical continuations are performed using DDE-Biftool. This is joint work with my PhD student J. Collera from Queen's University.

YUMING CHEN, Wilfrid Laurier University
Multiple epidemic waves in delayed SIR models on complex networks

We consider a delayed SIR epidemic model on an uncorrelated complex network and address the effect of time lag on the shape and multiple waves of epidemic curves. We show that when the transmission rate is above a threshold, large delay can cause multiple waves with larger amplitudes in the second and subsequent waves. This is a joint work with Dr. Shaofen Zou and Professor Jianhong Wu.

TONY HUMPHRIES, McGill University
Periodic Solutions of a Singularly Perturbed State Dependent DDE

We consider the singularly perturbed delay differential equation (DDE)

$$\varepsilon \dot{u}(t) = -\gamma u(t) - \sum_{i=1}^{2} \kappa_i u(t - a_i - c u(t)),$$

which has two linearly state-dependent delays. We show how to construct periodic solutions in the singular limit using geometric arguments. This allows us to construct the bifurcation diagram in the singular limit. We show that the bifurcation structures persist for small ε by studying the bifurcation structures numerically in that case. We find fold bifurcations and resulting regions of bistability on the principal branch. We also show that interaction between the delay terms determines the shape of the periodic solutions and branches. Finally, we show how the alignment of the branches of periodic solutions depends on the ratio a_2/a_1 of the delays at the trivial solution $u = 0$.

VICTOR LEBLANC, University of Ottawa
Realization of multiple Hopf bifurcations in DDEs

We will present some results (in collaboration with PL Buono and YS Choi) on the number of independent delay terms necessary in a DDE in order to realize the normal form for a multiple non-resonant Hopf bifurcation.

XINZHI LIU, University of Waterloo
Global convergence of neural networks with mixed time-varying delays

This talk discusses the dynamical behavior of a class of delayed neural networks with discontinuous neuron activations and general mixed time-delays involving both time-varying delays and distributed delays. Due to the presence of time-varying delays and distributed delays, the step-by-step construction of local solutions cannot be applied. This difficulty can be overcome by constructing a sequence of solutions to delayed dynamical systems with high-slope activations and show that this sequence converges to a desired Filippov solution of the discontinuous delayed neural networks. We then derive two sets of sufficient conditions for the global exponential stability and convergence of the neural networks, in terms of linear matrix inequalities (LMIs) and M-matrix properties (equivalently, some diagonally dominant conditions), respectively. Convergence behavior of both the neuron state and the neuron output are discussed. The obtained results extend previous work on global stability of delayed neural networks with Lipschitz continuous neuron activations, and neural networks with discontinuous neuron activations and only constant delays.

LIN WANG, University of New Brunswick
Influence of temporary migration on the transmission of infectious diseases in migrants' home residence

In this talk we propose a delay differential equation model to study the influence of temporary migration on the transmission of infectious diseases in migrants' home residence. The model is shown to admit a unique equilibrium which is locally asymptotically stable and is globally asymptotically stable under certain conditions. This implies that the disease always persists. Considering tuberculosis as an example, we explore various disease prevention and control strategies numerically to demonstrate how the migration related parameters affect the early outbreak of the disease. We find that a single control strategy such as reducing the migration time period alone has little effect on reducing the disease endemic level. For disease prevention and control, temporary migrant workers should be identified as the top target group and a combination of several prevention strategies should be implemented.

GAIL WOLKOWICZ, McMaster University
Predator-prey models with time delay in the conversion process

The dynamics of the classical predator-prey model and the predator-prey model based in the chemostat are studied and compared to see whether delay in the conversion process can lead to sustained oscillatory behaviour, when no such behaviour is possible when delay is ignored.

JIANHONG WU, Centre for Disease Modelling, York University
Seasonal migration dynamics and finite dimensional reduction of periodic delay differential systems

This is based on joint work with Xiang-Sheng Wang. We study a patchy model of bird migration between the summer breeding ground and the winter refuge site, along a finite set of stopovers. The model involves time lags for the transition time between the patches, and the model parameters are periodic in time as the biological activities related to the migration and reproduction are seasonal. Based on a surprising observation that the delay differential system is completely characterized by a finite dimensional ODE system, we construct a finite dimensional map to capture the bird migration dynamics. As a consequence, we derive the threshold condition, explicitly in terms of the model parameters, for the extinction and persistence of the considered bird species.

PEI YU, University of Western Ontario
Bifurcation Analysis on an HIV-1 Model with Time Delays

In this talk, we present a more realistic model for HIV-1 infection with two time delays, one for the average latent period of cell infection and the other for the average time needed for the virus production after a virion enters a cell. It is shown that, similar to the model without time delays, as the reproduction number of the system is varied, the model undergoes a series of bifurcations through infection-free equilibrium, single-infection equilibrium and double-infection equilibrium, leading to a Hopf or double-Hopf bifurcation. We also show that only the model with time delays may have double-Hopf bifurcation. Evidences indicate that larger delays may be able to help eradicate the virus.

XIAOQIANG ZHAO, Memorial University of Newfoundland
Some Remarks on Periodic Solutions for Functional Differential Equations

In this talk, I will show how the theory of global attractors and steady states for uniformly persistent dynamical systems can be used to obtain the existence of positive periodic solutions for dissipative periodic functional differential equations of retarded and neutral type. This result is then applied to a multi-species competition system and some epidemic models. A note on the weak compactness of solution maps will also be given.