Composition Operators
Opérateurs de composition
(Org: **Javad Mashreghi** (Laval) and/et **Nina Zorboska** (Manitoba))

CARL COWEN, I U P U I

Spectral Picture for Some Composition Operators

If ϕ is an analytic map of the disk into itself and H is a Hilbert space of analytic functions on the disk, the composition operator C_ϕ is the operator given by $C_\phi f = f \circ \phi$ for f in H. In this talk, we will discuss the point spectrum of C_ϕ^* on H^2 when $\phi(0) = \phi'(0) = 0$ or more generally, when ϕ has a fixed point in the open disk, but ϕ is not locally univalent there. The (power-)compact case is easy:

$$\sigma(C_\phi^*) = \sigma_p(C_\phi^*) = \{0, 1\}$$

In her recent thesis, Maria Neophytou used work of Poggi-Corradini and the speaker to show that for a broad class of such composition operators, the point spectrum of C_ϕ^* is an open disk centered at the origin with radius $1/\sqrt{\phi'(b)}$ where b is a fixed point of ϕ on the unit circle such that

$$\phi'(b) = \min\{\phi'(c) : c \text{ is a fixed point of } \phi \text{ with } |c| = 1\}$$

Her proof will be outlined and a conjecture will be offered.

PAUL GAUTHIER, Université de Montréal

Hypercyclicity of the Riemann zeta-function for the composition operator $C_i(f)(z) = f(z + i)$.

For a non-zero complex number a let $\phi_a(z) = z+a$ and C_a be the associated composition operator $C_a(f) = f \circ \phi_a$. The operator C_a is hypercylic on the space of entire functions. That is, there is an entire function f such that $\{C_a^n(f) : n \geq 0\}$ is dense in the space of entire functions (Birkhoff). Bagchi has shown that the Riemann zeta-function is in fact frequently hypercyclic for the vertical composition operator C_i on the space of zero-free holomorphic functions in the strip $\{z : 1/2 < \Re z < 1\}$. Removing the zero-free hypothesis violates the Riemann hypothesis. We discuss a recent observation by J. Andersson relating these questions to an interesting question on polynomial approximation.

GORDON MACDONALD, University of Prince Edward Island

Composition Operators on the Newton Space

We investigate properties of composition operators C_ϕ on the Newton space (the Hilbert space of analytic functions which have the Newton polynomials as an orthonormal basis). We derive a formula for the entries of the matrix of C_ϕ with respect to the basis of Newton polynomials in terms of the value of the symbol ϕ at the non-negative integers. We also establish conditions on the symbol ϕ for boundedness, compactness, and self-adjointness of the induced composition operator C_ϕ. A key technique in obtaining these results is use of an isomorphism between the Newton space and the Hardy space via the Binomial Theorem.

(Joint work with Peter Rosenthal.)

JAVAD MASHREGHI, Universite Laval

Composition operators on subspaces of H^2

Let b be in the closed unit ball of H^∞. In this case, the Toeplitz operator T_b is simply the multiplication by b on H^2. Then the $\mathcal{H}(b)$ space is defined to be the image of operator $(Id - T_b T_b^*)^{1/2}$, endowed with the scalar product

$$\langle (Id - T_b T_b^*)^{1/2} f, \, (Id - T_b T_b^*)^{1/2} g \rangle_{\mathcal{H}(b)} = \langle f, g \rangle_{H^2},$$

where $f, g \in (\text{ker}(Id - T_b T_b^*)^{1/2})^\perp$. This is a Hilbert space which is contractively contained in H^2. If $b = \Theta$ is inner, then $\mathcal{H}(\Theta)$ is a closed subspace of H^2 and is usually denoted by K_Θ. We will discuss the composition operators C_φ, with an inner symbol φ, on model subspaces K_Θ. If φ is an automorphism of the open unit disc, we also show that C_φ maps $\mathcal{H}(b)$, with b non-extreme, onto $\mathcal{H}(b \circ \varphi)$.

GABRIEL PRAJITURA, SUNY Brockport

Chaotic behavior of composition operators

There are at least 5 definitions currently in use for chaotic behavior. Not only that they are not equivalent but in the general topological setting they are not even related. Nevertheless, in the case on Hilbert space operators some of these notions turn out to be equivalent. In the particular case of composition operators on the Hardy space there are more such equivalences.

KATIE QUERTERMOUS, James Madison University

Composition and Toeplitz-Composition C-algebras Related to Linear-fractional Maps*

Let φ be an analytic self-map of the unit disk $\mathbb{D}$, and let $H^2(\mathbb{D})$ denote the Hardy space of the disk. We define the composition operator C_φ by $C_\varphi f = f \circ \varphi$ for all $f \in H^2(\mathbb{D})$. We are particularly interested in composition operators induced by linear-fractional, non-automorphism self-maps of $\mathbb{D}$ that fix a given point ζ on the unit circle and satisfy $\varphi'(\zeta) \neq 1$.

In this talk, we consider two types of composition C*-algebras: $C^*(C_\varphi, \mathcal{K})$, the unital C*-algebra generated by the ideal of compact operators and a single linear-fractionally-induced composition operator of the form described above, and $C^*(\mathcal{F}_\zeta)$, the unital C*-algebra generated by the collection of all composition operators induced by linear-fractional non-automorphisms that fix a given point ζ on the unit circle. We show that each of these C*-algebras is isomorphic, modulo the ideal of compact operators, to the unitization of an appropriate crossed product C*-algebra. We then determine the K-theory of $C^*(C_\varphi, \mathcal{K})$ and calculate the essential spectra of a class of operators in this C*-algebra.

We also investigate the Toeplitz-composition C*-algebra $C^*(T_z, C_\varphi)$, where T_z denotes the unilateral shift on $H^2(\mathbb{D})$. By combining our results with related results in the work of Jury and Kriete, MacCluer, and Moorhouse, we obtain a description of the structure of $C^*(T_z, C_\varphi)/\mathcal{K}$ for any linear-fractional self-map φ of $\mathbb{D}$.

MARIA TJANI, University of Arkansas

Closed-Range Composition Operators on Weighted Dirichlet spaces

For an analytic self-map of the unit disk $\mathbb{D}$ we give new necessary and sufficient conditions for the composition operator C_ϕ to be closed-range on Dirichlet type spaces. An important ingredient is a theorem due to Luecking.

WEN XU, Memorial University of Newfoundland

Composition operators from Bloch type spaces into Q_K type spaces

Bounded and compact composition operators C_φ, induced by an analytic self-map of the open unit disc $\mathbb{D}$ of the complex plane $\mathbb{C}$, are characterized from the Bloch-type spaces $\mathcal{B}_\mu$ and $\mathcal{B}_{\mu,0}$ into the Q_K type spaces $Q_K(p, q)$.

RUHAN ZHAO, SUNY-Brockport

New estimates of essential norms of weighted composition operators between Bloch type spaces

For $\alpha > 0$, the α-Bloch space is the space of all analytic functions f on the unit disk D satisfying

$$\|f\|_{B^\alpha} = \sup_{z \in D} |f'(z)|(1 - |z|^2)^\alpha < \infty.$$

Let φ be an analytic self-map of D and u be an analytic function on D. The weighted composition operator induced by u and φ is defined by $uC_\varphi(f)(z) = u(z)f(\varphi(z))$. We give estimates of the essential norms of uC_φ between different α-Bloch spaces in terms of the n-th power of φ. We also give similar characterizations for boundedness and compactness of uC_φ between different α-Bloch spaces. This is a joint work with Jasbir Singh Manhas.