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Composition and Toeplitz-Composition C*-algebras Related to Linear-fractional Maps

Let $\varphi$ be an analytic self-map of the unit disk $\mathbb{D}$, and let $H^2(\mathbb{D})$ denote the Hardy space of the disk. We define the composition operator $C_\varphi$ by $C_\varphi f = f \circ \varphi$ for all $f \in H^2(\mathbb{D})$. We are particularly interested in composition operators induced by linear-fractional, non-automorphism self-maps of $\mathbb{D}$ that fix a given point $\zeta$ on the unit circle and satisfy $\varphi'(\zeta) \neq 1$.

In this talk, we consider two types of composition C*-algebras: $C^*(C_\varphi, K)$, the unital C*-algebra generated by the ideal of compact operators and a single linear-fractionally-induced composition operator of the form described above, and $C^*(F_\zeta)$, the unital C*-algebra generated by the collection of all composition operators induced by linear-fractional non-automorphisms that fix a given point $\zeta$ on the unit circle. We show that each of these C*-algebras is isomorphic, modulo the ideal of compact operators, to the unitization of an appropriate crossed product C*-algebra. We then determine the K-theory of $C^*(C_\varphi, K)$ and calculate the essential spectra of a class of operators in this C*-algebra.

We also investigate the Toeplitz-composition C*-algebra $C^*(T_z, C_\varphi)$, where $T_z$ denotes the unilateral shift on $H^2(\mathbb{D})$. By combining our results with related results in the work of Jury and Kriete, MacCluer, and Moorhouse, we obtain a description of the structure of $C^*(T_z, C_\varphi)/K$ for any linear-fractional self-map $\varphi$ of $\mathbb{D}$. 