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An irrationality measure for Mahler numbers

Let $F(x) \in \mathbb{Z}[[x]]$ be a power series that satisfies a Mahler-type functional equation; that is, there exist positive integers k and d and polynomials $p(x), a_0(x), \dots, a_d(x) \in \mathbb{Z}[x]$ with $a_0(x)a_d(x) \neq 0$ such that

$$p(x) + \sum_{i=0}^d a_i(x)F(x^{k^i}) = 0.$$

Let ξ be a real number. The *irrationality exponent* $\mu(\xi)$ of ξ is defined as the supremum of the set of real numbers μ such that the inequality $|\xi - p/q| < q^{-\mu}$ has infinitely many solutions $(p, q) \in \mathbb{Z} \times \mathbb{N}$.

In this talk we will outline a proof that $\mu(F(a/b)) < \infty$ for all positive integers $b \geq 2$ such that a/b is in the radius of convergence of $F(x)$ and $\log |a|/\log b \in [0, 1/2)$; in particular, we show that $F(1/b)$ is not a Liouville number. This generalizes a result of Adamczewski and Cassaigne for automatic numbers.

This is joint work with Jason Bell.