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On rational approximation to real points on conics

Let $q(x_0, x_1, x_2)$ be a homogeneous polynomial of degree 2 in 3 variables, with rational coefficients. Assume that q admits a non-trivial real zero and that q is irreducible over the field $\mathbb{Q}$ of rational numbers. Denote by U the set of real zeros of q having $\mathbb{Q}$-linearly independent coordinates. We show that

a) each point in U has an exponent of uniform rational approximation between $1/2$ and $1/\gamma \cong 0.618$, where γ denotes the golden ratio,

b) the elements of U for which the upper bound is achieved form an infinite countable set.

For $q(x_0, x_1, x_2) = x_0x_2 - x_1^2$, the statement a) is due to Davenport and Schmidt (1967) while b) is due to the author (2003). When q is irreducible over $\mathbb{R}$ and admits a non-trivial rational zero, we are quickly reduced to that case. Otherwise, the proof of a) is simpler, but the existence of "extremal" points in b) requires additional tools.