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Residual normal crossings of Schubert varieties

A divisor $D = \{f = 0\}$ in $\mathbb{A}^n$ is called **residual normal crossings** if $\operatorname{init} f = \prod_{i=1}^n x_i$, for some term order. From D we can build a stratification $\mathcal{Y}$ of $\mathbb{A}^n$ by varieties, by decomposing D into components, intersecting them, and repeating this process.

Theorem.

1. For each $Y \in \mathcal{Y}$, $\operatorname{init} Y$ is defined by *squarefree* monomials, i.e. is the Stanley-Reisner scheme of a simplicial complex.
2. init commutes with taking unions and intersections of strata $Y \in \mathcal{Y}$.
3. There is a natural surjection $2^n \to \mathcal{Y}$ of posets, that one can think of as giving a decomposition of the simplex Δ^{n-1} with strata indexed by $\mathcal{Y}$.
4. If Y's closed subcomplex of Δ^{n-1} is a shellable ball, then Y is Cohen-Macaulay. If Y's open subcomplex is the interior of that ball, then Y is normal.

Our principal example is when $\mathbb{A}^n$ is an opposite Bruhat cell $X^v_\circ$ in a (possibly infinite-dimensional) flag manifold, and $\mathcal{Y}$ is induced from the Bruhat decomposition. Then the above theorem recovers the facts that Schubert varieties are normal and Cohen-Macaulay.

If time permits, I'll talk about varieties + stratifications that are covered by an atlas of opposite Bruhat cells, such as the Grassmannian with the Lusztig-Postnikov stratification [Snider], and (conjecturally) the wonderful compactification of a group [He-K-Lu, in preparation].