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Monotone Hurwitz numbers and the HCIZ integral

A remarkable fact discovered by theoretical physicists is that certain integrals over N by N matrices admit large N asymptotic expansions whose coefficients are generating functions enumerating maps on compact surfaces. In certain cases these matrix integrals can be evaluated, and one gets enumerative formulas, e.g. for planar maps. A generalization of this theme involves integrating over pairs of interacting matrices - this corresponds to counting coloured maps. Unfortunately, one encounters a difficult obstruction to solving multimatrix integrals. This obstruction is yet another matrix integral known as the Harish-Chandra-Itzykson-Zuber integral. Physicists have conjectured that the HCIZ integral itself admits an asymptotic expansion for "something combinatorial," but this something was unknown. I will discuss recent work with Ian Goulden and Mathieu Guay-Paquet in which we show that the asymptotic orders of the HCIZ integral are generating functions for a desymmetrized variant of Hurwitz numbers, which we call "monotone Hurwitz numbers." The resulting combinatorics can be treated by suitably augmenting the combinatorial approach to Hurwitz theory, and in this way one can get analytic results on the HCIZ integral by solving combinatorial problems.