LARGE CARDINALS AND PRIKRY TYPE FROCING

The singular cardinal problem is the project to find a complete set of rules for the behavior of the function $\kappa \mapsto 2^\kappa$ for singular cardinals κ. A major tool is Prikry type forcing involving large cardinals. The purpose of this course is to describe various Prikry type notions of forcing and results on the possible values of the power set function for singular cardinals.

Outline:

(1) Preliminaries on forcing and large cardinals (we will define measurable cardinals, supercompact cardinals. strong cardinals, Mitchell order)
(2) The classical Prikry forcing: if κ is a measurable cardinal, then there is a generic extension in which cardinals are preserved and the cofinality of κ is ω.
(3) Magidor's Forcing: generalizing the above result to arbitrary cofinalities. More precisely, for a regular λ, Magidor forcing adds a club set of order type λ in κ, starting with a Mitchell order increasing sequence $\langle U_\alpha \mid \alpha < \lambda \rangle$ of normal measures on κ.
(4) Supercompact Prikry forcing: this forcing adds an increasing ω-sequence of sets $x_n \in (\mathcal{P}_\kappa(\eta))^V$ with $\eta = \bigcup_n x_n$, starting from a supercompactness measure U on $\mathcal{P}_\kappa(\eta)$.
(5) Extender based Prikry forcing: start with ω many strong cardinals and blow up the powerset of their supremum without adding bounded subsets to it.
(6) Interleaving collapses: combine the above forcings with collapses in order to push down the constructions to smaller cardinals.
(7) Some open problems.