Symbolic Dynamics and Ergodic Theory
Dynamique symbolique et théorie ergodique

(Org: **Chris Bose** (Victoria), **Doug Lind** (Washington), **Ian Putnam** (Victoria) and/et **Anthony Quas** (Victoria))

MAHSA ALLAHBAKHSHI, University of Victoria

Uniform Conditional Distribution and Class Degree

The degree of a code π where π is a factor code on a shift of finite type is defined the number of preimages of a "typical" point when π is finite to one. We generalize the concept of the degree to the class degree which is applied to any factor code defined on a shift of finite type. By showing the uniform conditional distribution property for measures of relative maximal entropy, it is shown that the class degree is an invariant upper bound on the number of such measures.

MARCY BARGE, Montana State University

Maximal equicontinuous factors of Pisot family tiling spaces

We show that the maximal equicontinuous factor of the $\mathbb{R}^n$-action on an n-dimensional Pisot family substitution tiling space is a Kronecker action on an nd-dimensional solenoid, d being the algebraic degree of the eigenvalues of the inflation. The map onto the maximal equicontinuous factor is finite-to-one and a.e. m-to-one for some m: we prove that $m = 1$ if and only if the proximal relation for the $\mathbb{R}^n$-action is closed. Thus the $\mathbb{R}^n$-action has pure discrete spectrum if and only if its proximal relation is closed.

We also show that the pull back of the map onto the maximal equicontinuous factor is injective on 1st integer cohomology with image identifiable as the group of eigenvalues of the tiling flow.

Consideration of the maximal equicontinuous factor leads to the following characterization of minimal directions: If the tiling space is n-dimensional and self-similar Pisot, then the $\mathbb{R}$-action $(T, t) \mapsto T - tv$ is not minimal on the tiling space if and only if v lies in a proper subspace of $\mathbb{R}^n$ spanned by return vectors. In particular, the set of minimal directions is a full measure G_δ with dense complement.

ARNO BERGER, University of Alberta

Some dynamical properties of Benford sequences

Numerical data generated by dynamical processes often obey Benford's Law (BL) of logarithmic mantissa distributions. For nonautnomous processes $x_n = T_n(x_{n-1})$, this talk will present some necessary as well as sufficient conditions for (x_n) to conform with BL in its strongest form. The assumptions on (T_n), viz. asymptotic convexity and eventual expansivity on average, are very mild and met e.g. by many rational or exponential maps, and combinations thereof. Some interesting open problems will be mentioned.

CHRIS BOSE, University of Victoria

An amateur's guide to the ergodic theory of nonuniformly hyperbolic systems

Using one or two simple, explicit examples (generalized baker's maps, in fact) we will describe some key ideas and recent developments in this field.

SASHA FISH, University of Wisconsin-Madison

Ergodic Plunnecke inequalities with applications

We prove an ergodic version of Plunnecke inequalities and give a combinatorial application of them. Based on a joint work with M.Bjorklund (ETH,Zurich)

BRADY KILLOUGH, Mount Royal University
A construction of the Bowen measure from heteroclinic points

We consider the class of irreducible hyperbolic dynamical systems known as irreducible Smale spaces. In a Smale space the dynamics give rise to notions of stable and unstable equivalence. Furthermore, for each such space there is an unique invariant probability measure that maximizes the entropy (the Bowen measure). Bowen constructed this measure as a limit of measures supported on periodic points. We provide an alternative construction as a limit of measures supported on heteroclinic points (points that are stably equivalent to some given point, and unstably equivalent to another given point). Our proof relies on properties of shifts of finite type, and resolving factor maps. This is joint work with Ian Putnam.

JAREK KWAPISZ, Montana State University
Rigidity and Mapping Class Group for Self-Similar Tiling Spaces

For translation actions on tiling spaces associated to self-similar tilings, we show that existence of a homeomorphism between the spaces implies conjugacy of the actions up to a linear rescaling. We also introduce the general linear group of a tiling, prove its discreteness, and show that it is naturally isomorphic with the (pointed) mapping class group of the tiling space. To illustrate our theory, we compute the mapping class group for a five-fold symmetric Penrose tiling.

DOUG LIND, University of Washington
Principal Actions of Discrete Groups

I will discuss joint work with Klaus Schmidt which investigates principal algebraic actions of a discrete countable group Γ. These are defined using the principal ideal generated by an element f of the integral group ring $\mathbb{Z}[\Gamma]$. We are able give at least partial answers some important dynamical questions, such as ergodicity, expansiveness, and entropy, using machinery such as the Fuglede-Kadison determinants from von Neumann algebras. There are still many fascinating open problems.

BRIAN MARCUS, University of British Columbia
Limiting Entropy and Independence Entropy of d-dimensional shift spaces I

Topological entropy is a fundamental invariant for Z^d-shift spaces. When $d = 1$ and the shift space is a shift of finite type (SFT), it is easy to compute the topological entropy. In higher dimensions, it is much more difficult to compute, and exact values are known in only a handful of cases. However, a limiting entropy, as the dimension $d \to \infty$, defined as follows, can be computed in many more cases.

A 1-dimensional shift space X determines Z^d-shift spaces $X^{\otimes d}$ for each d; namely, $X^{\otimes d}$ is the Z^d-shift space where every "row" in every coordinate direction satisfies the constraints of X. Let $h_d(X)$ denote the topological entropy of $X^{\otimes d}$. Then $h_d(X)$ is non-increasing in d and its limit is denoted $h_\infty(X)$.

We introduce a notion of "independence entropy" denoted $h_{ind}(X)$, which is explicitly computable if X is a 1-dimensional SFT. We show that $h_\infty(X) \geq h_{ind}(X) = h_{ind}(X^{\otimes d})$ for all d. We can prove that equality holds in many cases, and it may well hold in general.

(joint work with Erez Louidor and Ronnie Pavlov)

TOM MEYEROVITCH, UBC and PIMS
Gibbs and equilibrium measures for some families of subshifts

Consider a d-dimensional subshift of finite type X and $f : X \to \mathbb{R}$ with d-summable variation. A theorem of Lanford and Ruelle states that any equilibrium measure is Gibbs. A partial converse is given by Dobrusin's theorem : If X is a strongly-irreducible subshift, Gibbs-measures are equilibrium measures. I will discuss extensions and generalizations of these theorems for classes subshifts of infinite type.

ROBERT MOODY, University of Victoria
Ergodic dynamical systems in the study of diffraction

Given a measure, putatively the diffraction of something, how does one create such a something? In this talk we show what a general solution to this problem looks like in the case of pure point diffraction, and what sort of problems it brings with it. The result may be viewed as an elaboration of the classical Halmos - von Neumann theorem on pure point dynamical systems. The results are joint work with Daniel Lenz.

DAVE MORRIS, University of Lethbridge
When do subsets of $\{0,1\}^{G\times G}$ contain recurrent points?

Suppose G is a (countable) group and C is a closed, nonempty, G-invariant subset of $\{0,1\}^{G\times G}$. For an application in the theory of orderable groups, we would like to know there exists a point $c \in C$, such that c is recurrent for the action of each $T \in G$.

The Poincaré Recurrence Theorem implies this is the case when there is a G-invariant probability measure on C, so c always exists if G is abelian, or, more generally, if G is amenable. For what other classes of groups does such a point c always exist? In particular, does it suffice to assume that G has no nonabelian free subgroups?

VIOREL NITICA, West Chester University
Transitivity of Heisenberg group extensions

We show that among C^r extensions ($r > 0$) of a uniformly hyperbolic dynamical system with fiber the standard real Heisenberg group H_n of dimension $2n+1$, those that avoid an obvious obstruction to topological transitivity are generically topological transitive. Moreover, if one considers extensions with fiber a connected nilpotent Lie group with compact commutator subgroup (for example, H_n/Z), among those that avoid the obvious obstruction, topological transitivity is open and dense. This is joint work with I. Melbourne and A. Torok.

PIOTR OPROCHA, AGH University of Science and Technology, Poland
On distributional chaos and subshifts

Distributional chaos was introduced in 1994 by Schweizer and Smítal. The strongest version of this property is defined in the following way.

Let $f\colon X \to X$ be a continuous map acting on a compact metric space (X,ρ). For any positive integer n, points $x,y \in X$ and $t \in \mathbb{R}$ let

$$\Phi_{xy}^{(n)}(t) = \frac{1}{n}\left|\{i \,:\, \rho(f^i(x), f^i(y)) < t \quad, 0 \le i < n\}\right|$$

where $|A|$ denotes the cardinality of the set A. Denote by Φ_{xy} and Φ_{xy}^* the following functions

$$\Phi_{xy}(t) = \liminf_{n\to\infty} \Phi_{xy}^{(n)}(t) \quad , \quad \Phi_{xy}^*(t) = \limsup_{n\to\infty} \Phi_{xy}^{(n)}(t).$$

If there is an uncountable set $S \subset X$ so that $\Phi_{xy}(s) = 0$ for some $s > 0$ and $\Phi_{xy}^*(t) = 1$ for all $t > 0$, provided that $x, y \in S$, $x \neq y$, then we say that f is distributionally chaotic.

Many interesting results on distributional chaos were obtained via tools of symbolic dynamics. In this talk we will survey some recent results and state a few open problems.

RONNIE PAVLOV, University of Denver
Limiting Entropy and Independence Entropy of d-dimensional shift spaces II

We present further work on the relationship between the concepts of independence entropy ($h_{ind}(X)$) and limiting d-dimensional entropy ($h_\infty(X)$) presented in Brian Marcus's talk. In particular, we prove that $h_{ind}(X) = h_\infty(X)$ whenever X is a nearest-neighbor shift of finite type. As a corollary, this shows that a simple closed form can be found for the limiting entropy $h_\infty(X)$ when X is a nearest-neighbor SFT. We also show that if $h_{ind}(X) = h_\infty(X)$ for all SFTs, then the same is true for general shift spaces.

MARCUS PIVATO, Trent University
Positive expansiveness versus network dimension in symbolic dynamical systems

Any 'symbolic dynamical system' can be seen as a continuous transformation $\Phi : \mathcal{X} \longrightarrow \mathcal{X}$ of a closed subset $\mathcal{X} \subseteq \mathcal{A}^{\mathbb{V}}$, where $\mathcal{A}$ is a finite 'alphabet' and $\mathbb{V}$ is a countable indexing set. (Examples include subshifts, odometers, cellular automata, and automaton networks.) The function Φ induces a directed graph (or 'network') structure on $\mathbb{V}$. (For example: the network of a cellular automaton is a Cayley digraph.) The geometry of this network reveals information about the dynamical system $(\mathcal{X}, \Phi)$. The *dimension* $\dim(\mathbb{V})$ is an exponent describing the growth rate of balls in this network as a function of their radius. We show: if $\mathcal{X}$ has positive entropy and $\dim(\mathbb{V}) > 1$, and the system $(\mathcal{A}^{\mathbb{V}}, \mathcal{X}, \Phi)$ satisfies minimal symmetry and mixing conditions, then $(\mathcal{X}, \Phi)$ cannot be positively expansive. This generalizes a well-known result of Shereshevsky about multidimensional cellular automata. We also construct a counterexample to a version of this result without the symmetry condition. Finally, we show that network dimension is invariant under topological conjugacies which are Hölder-continuous.

ANTHONY QUAS, Anthony Quas
Semi-invertible Multiplicative Ergodic Theorems

We discuss the semi-invertible case of the Oseledets multiplicative ergodic theorem (when the underlying transformation is invertible but the matrices being multiplied are not necessarily). We give a substantial strengthening of the theorem's conclusion in this case and consider applications in ocean models. We also discuss extensions to the case of composition of operators.

Joint work with Gary Froyland and Simon Lloyd.

CECILIA GONZALEZ TOKMAN, University of Victoria
ACIMs for metastable systems

Metastable systems arise when a dynamical system possessing two or more invariant sets of positive Lebesgue measure is perturbed in a way that the initially invariant sets merge.

Working in a one-dimensional setting of piecewise smooth expanding maps, we describe the limit(s), as the size of the perturbation goes to zero, of absolutely continuous invariant measures (ACIMs) for metastable systems as an explicit convex combination of the ergodic ACIMs for the initial system.

This talk is based on joint work with B. Hunt and P. Wright.

REEM YASSAWI, Trent University and McGill University
Adic representations of one sided substitution subshifts

Given a substitution τ defined on an alphabet $\mathcal{A}$, we can consider either the 1-sided or 2-sided subshifts generated by τ, denoted $(X_\tau^{\mathbb{N}}, \sigma)$, and $(X_\tau^{\mathbb{Z}}, \sigma)$ respectively. We show that for a large class of substitutions τ, $(X_\tau^{\mathbb{N}}, \sigma)$ has a representation as an adic system. We also show by example that conjugacy of the 2-sided substitution subshifts $(X_{\tau_1}^{\mathbb{Z}}, \sigma)$ and $(X_{\tau_2}^{\mathbb{Z}}, \sigma)$ does not imply conjugacy of $(X_{\tau_1}^{\mathbb{N}}, \sigma)$ and $(X_{\tau_2}^{\mathbb{N}}, \sigma)$, and discuss conditions on the substitutions so that 2-sided conjugacy would imply 1-sided conjugacy.