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The Favard length of product Cantor set with tiling condition

Favard length is defined as the average length of the 1-dimensional projection of a set in \mathbf{R}^2 . It comes from the so called Buffon's needle problem in Geometric probability. Besicovitch stated in his famous projection theorem that the Favard length of an irregular set of Hausdorff dimension 1 is zero. It has long been of interest to mathematician since then that how fast can the Favard length of finite iterations of self-similar sets go to zero. The first quantitative upper bound was due to Peres and Solomyak. They proved that the Favard length of self-similar set decays faster than an exponential function with some iterated logarithm as exponent, which is weak compared with the conjecture rate n^{-1} . In 2008, Nazarov, Peres and Volberg (NPV) got a power type upper bound for 4-corner Cantor set. In a joint project with my supervisor Izabella Łaba, we extended their result to a wider class of self-similar product sets with the tiling condition, that is, there exists a direction such that the projection onto it has positive Lebesgue measure. Following Bateman and Volberg's argument, it turns out that for a 1-Hausdorff dimensional purely unrectifiable self-similar set with tiling condition its Favard length decays slower than $\frac{\log n}{n}$, where n represents the n -th iteration of the self-similar set.