Analysis and Geometry of Nonlinear Partial Differential Equations
Analyse et géométrie des équations aux dérivées partielles non linéaires
(Org: **Nassif Ghoussoub** (UBC), **Young-Heon Kim** (UBC) and/et **Robert McCann** (Toronto))

MARTIAL AGUEH, University of Victoria
Barycenters in the Wasserstein space

We introduce a notion of barycenter in the Wasserstein space which generalizes the interpolation of McCann to the case of more than two probability measures. We discuss existence, uniqueness, characterization and regularity of the barycenter, and relate it to the multi-marginal Monge-Kantorovich problem. We then introduce a notion of "barycentric convexity" (i.e. convexity along barycenters) in the Wasserstein space which generalizes the notion of displacement convexity, and we give some examples.

LIA BRONSARD, McMaster University
Global minimizers for anisotropic superconductors

SUN-SIG BYUN, Seoul National University
Hessian estimates for fourth-order elliptic systems with measurable coefficients

We consider a fourth-order elliptic system with measurable coefficients in a nonsmooth domain to find a minimal regularity assumption on the coefficients and a lower level of geometric structure on the boundary of the bounded domain for a classical $W^{2,p}$-regularity theory.

RUSTUM CHOKSI, McGill University
Asymptotic Analysis of the Dilute Regime in Phase Field Models

Via an asymptotic development of the energy, we explore the small volume fraction regime associated with phase field models containing long-range interactions. Using the language of Gamma-convergence, we describe both the leading order behavior — associated with coarsening of particles, and the next-order behavior — associated with self-oganiziation of particles. This is joint work with M. Peletier (TU Eindhoven).

We also discuss work in progress with N. Le (Columbia) and Peletier which exploits the Gamma-limit structure of the energy to prove convergence of the associated gradient flows. In particular, we connect to the well-known LSW theory for Ostwald ripening.

MARINA CHUGUNOVA, University of Toronto
Convergence to equilibrium for a non-wellposed thin film equation

We study a lubrication model of the time-evolution of a thin liquid film on a horizontal stationary cylinder. The PDE equation defines a generalized gradient flow. We prove that for each given mass there is a unique steady state, which minimizes the energy and attracts all global strong solutions with finite-entropy initial values. The steady state is symmetrically decreasing, supported on a compact subinterval, and meets the dry region at a zero contact angle. We show that the distance of any solution from the steady state cannot decrease faster than a power law. The proofs rely on new applications of energy and entropy inequalities.

Joint work with Almut Burchard and Benjamin Stephens

JAYWAN CHUNG, KAIST (Korea Advanced Institute of Science and Technology)
Long-time asymptotics of the zero level set for the heat equation

The zero level set $Z(t) := \{\mathbf{x} \in \mathbf{R}^d : u(\mathbf{x}, t) = 0\}$ of a solution u to the heat equation in $\mathbf{R}^d$ is considered. Under vanishing conditions on moments of the initial data, we will prove that the set $Z(t)$ in a ball of radius $C\sqrt{t}$ for any $C > 0$ converges to a specific graph as $t \to \infty$ when the set is divided by $\sqrt{t}$. Solving a linear combination of the Hermite polynomials gives the graph and coefficients of the linear combination depend on moments of the initial data. Also the graphs to which the zero level set $Z(t)$ converges can be classified in some cases.

PENGFEI GUAN, McGill University
Curvature measures, isoperimetric inequalities and fully nonlinear PDE

We discuss the connections of elliptic and parabolic fully nonlinear partial differential equations with curvature measures and isoperimetric inequalities for the quermassintegrals. We settle the prescirbing curvature measure problem by solving a corresponding elliptic equation, with a new key estimate. We also propose some parabolic curvature flows to prove the isoperimetric type inequalities.

DAVID HARTENSTINE, Western Washington University
Asymptotic statistical characterizations of p-harmonic functions of two variables

It is well known that harmonic functions are characterized by the mean value property. Building on recent work of Manfredi, Parviainen and Rossi, we generalize this result, proving that a p-harmonic function of two variables satisfies, in a viscosity sense, two asymptotic formulas involving its local statistics. Moreover, we show that these asymptotic formulas characterize p-harmonic functions when $1 < p < \infty$. An example demonstrates that, in general, these formulas do not hold in a non-asymptotic sense. This is joint work with Matthew Rudd (University of the South).

WEIYONG HE, University of Oregon
The complex Monge-Ampere equation on compact Kaehler manifolds

We consider the complex Monge-Ampere equation on a compact Kaehler manifold (M, g) when the right hand side F has rather weak regularity. In particular we prove that estimate of $\triangle\phi$ and the gradient estimate hold when F is in W^{1,p_0} for any $p_0 > 2n$. As an application, we show that there exists a classical solution in W^{3,p_0} for the complex Monge-Ampere equation when F is in W^{1,p_0}.

SLIM IBRAHIM, Univeristy of Victoria
Global small solutions to the Navier-Stokes-Maxwell equations

We consider a full system of Magneto-Hydro-Dynamic equations. The system formally satisfies an energy estimate. Nevertheless, the existence of global weak solution seems to remain an interesting open problem in both two and three space dimension. In 3D, we show the existence of global small solutions (Kato-type). In 2D, we prove the same result in a space "close" to the energy space.

This is a joint work with S. Keraani (University of Lille 1, France)

ROBERT JERRARD, University of Toronto
Some remarks on binormal curvature flow

We study a class of weak solutions of the geometric evolution problem of binormal curvature flow. Classically, a smooth map $\gamma : S^1 \times [0, T] \to \mathbb{R}^3$ defines a binormal curvature flow if $|\gamma_s| = 1$ and $\gamma_t = \gamma_s \times \gamma_{ss}$. We define a class of weak, measure-theoretic solutions that need only have the regularity of integral currents, and we prove that if a weak solution in this sense agrees at time $t = 0$ with a smooth embedded curve, then it coincides with the corresponding smooth binormal curvature flow until the first time when that flow develops self-intersections. This is joint work with Didier Smets.

YONG JUNG KIM, KAIST

The Newtonian potential in a relative sense

The Newtonian potential is introduced in a relative sense for radial functions. In this way one may treat the potential theory for a larger class of functions in a unified manner for all dimensions $d \geq 1$. For example, Newton's Theorem is given in terms of relative potentials, which is a simpler statement for all dimensions. This relative potential is then used to obtain the L^1-convergence order $O(t^{-1})$ as $t \to \infty$ for radially symmetric solutions to the porous medium and fast diffusion equations. The technique is also applied to radial solutions of the p-Laplacian equations to obtain the same convergence order.

KI-AHM LEE,

Homogenization of Soft inclusions

In this talk, we are going to discuss the soft inclusions of periodically distributed materials with periodicity ϵ. We call soft inclusion when the diffusion coefficient is zero on the included subset. Since the diffusion coefficient is nonzero outside of inclusions, there will be an effective diffusion when the periodicity ϵ goes to zero. The corresponding equation considered at this talk will be a fully nonlinear equation in the perforated domain with Neumann data on the boundary of inclusions. The effective equation will be obtained by constructing global periodic solutions called correctors. On the other hand, it is not clear what the compatibility condition for the nonlinear equation with Neumann data. So we introduce the concept of the first corrector to find the compatibility condition and the second corrector to average out the oscillations. When the compatibility condition violated, the effect of Neumann data will dominate. So we consider smaller inclusions so that the effect from diffusion and the influence of Neumann data are balanced.

ADAM OBERMAN, Simon Fraser University

Numerical Methods for Geometric Elliptic Partial Differential Equations.

Geometric Partial Differential Equations (PDEs) can be used to describe, manipulate and construct shapes based on intrinsic geometric properties such as curvatures, volumes, and geodesic lengths. These equations arise in classical areas of mathematics (Ricci Flow, Surface Theory) and are useful in modern applications (Image Registration, Computer Animation).

In general these equations are considered too difficult to solve, which is why linearized models or other approximations are commonly used. Progress has recently been made in building solvers for a class of Geometric PDEs. These solvers naturally give better geometric results and, in some cases, are competitive in terms of cost with the simplified models.

In this talk I'll give examples of a few important geometric PDEs which can be solved using a numerical method called monotone finite difference schemes: Monge-Ampere, Convex Envelope, Infinity Laplace, and Mean Curvature.

These methods have been implemented for registration of Brain Images. For Surface Registration, the Infinity Laplace equation is used to match surfaces using geodesic lengths [Sapiro]. For Volume Registration, the Monge-Ampere equation is used to minimize distortion of volumes [Tannenbaum-Haker-Haber]. Convergent numerical schemes are important in these applications: bad discretizations lead to artificial singularities in the mappings.

Focussing in on the Monge-Ampere equation, I'll show how naive schemes can work well for smooth solutions, but break down in the singular case. This makes having a convergent scheme even more important. I'll present a convergent solver which which is fast: comparable to solving the Laplace equation a few times.

BRENDAN PASS, University of Toronto

The multi-marginal optimal transportation problem

We consider an optimal transportation problem with more than two marginals. We use a family of semi-Riemannian metrics derived from the mixed, second order partial derivatives of the cost function to provide upper bounds for the dimension of the support of the solution.

LOUIS P. SAUMIER, University of Victoria
An Efficient Numerical Algorithm for the L2 Optimal Transport Problem with Periodic Densities

We present a numerical method to solve the optimal transport problem with a quadratic cost when the source and target measures are periodic probability densities in any space dimension. This method relies on a numerical resolution of the corresponding Monge-Ampère equation and uses an existing Newton-like algorithm (introduced by Loeper and Rapetti in 2005) that is generalized to the case of a non uniform final density. The main idea consists of designing an iterative scheme where the fully nonlinear equation is approximated by a non-constant coefficient linear elliptic PDE that is discretized and solved at each iteration, in two different ways: a second order finite difference scheme and a fast Fourier transform (FFT) method. The FFT method, made possible thanks to a preconditioning step based on the coefficient-averaged equation, results in an overall O(P log P)-operations algorithm, where P is the number of discretization points. In particular, we employ fourth order finite differences to approximate the action of the densities on the solution iterates, which result in more accurate results without having to sacrifice the efficiency of the overall algorithm. We will state a result showing that the algorithm converges to the solution of the optimal transport problem, under suitable conditions on the densities, and then give the key ideas justifying it. Numerical experiments demonstrating the robustness and efficiency of the method on examples of image processing, including an application to multiple sclerosis (MS) disease detection, will also be presented. This is joint work with Martial Agueh and Boualem Khouider.

ARTUR SOWA, University of Saskatchewan
Nonlinear eigenvalue problems with applications to quantum systems

Differential equations with negative-power nonlinearities are encountered in physical models more and more often, most notably in the context of micro- and nano-systems. At the same time, this type of nonlinearity typically does not yield easily to the traditional methods of analysis. In this talk I will discuss nonlinear eigenvalue problems of the type

$$\lambda\, Df = f - \frac{1}{f},$$

where D stands for a linear operator. I will demonstrate that for a number of special operators D — e.g. one may take $D = \partial_s^2$, acting in the space of Dirichlet series — the stated problem can be adequately interpreted and fully resolved. In particular, I will discuss an algorithmic method for constructing the complete set of solutions. The resulting eigenfunction bases are applicable as a foundation for specialized signal processing, particularly in the context of magnetic quantum- and nano-systems. I will also point at some conceptual connections with the Number Theory. Finally, I will discuss opportunities for generalizing these results.

VITALI VOUGALTER, University of Toronto
On the solvability conditions for a linearized Cahn-Hilliard equation

We derive solvability conditions in $H^4(\mathbb{R}^3)$ for a fourth order partial differential equation which is a linearized Cahn-Hilliard problem using the results obtained for a Schroedinger type operator without Fredholm property in our preceding work.

JIE XIAO, Memorial University of Newfoundland
Regularity of Morrey potentials

This talk, based on a recent paper joint with David R. Adams, will discuss trace estimates for Riesz potential integrals of Morrey functions, and consequently, C^∞ smoothness of a weak solution to the generalized harmonic map equation $-\Delta u = |\nabla u|^p u$.