ADAM OBERMAN, Simon Fraser University
Numerical Methods for Geometric Elliptic Partial Differential Equations.

Geometric Partial Differential Equations (PDEs) can be used to describe, manipulate and construct shapes based on intrinsic geometric properties such as curvatures, volumes, and geodesic lengths. These equations arise in classical areas of mathematics (Ricci Flow, Surface Theory) and are useful in modern applications (Image Registration, Computer Animation).

In general these equations are considered too difficult to solve, which is why linearized models or other approximations are commonly used. Progress has recently been made in building solvers for a class of Geometric PDEs. These solvers naturally give better geometric results and, in some cases, are competitive in terms of cost with the simplified models.

In this talk I'll give examples of a few important geometric PDEs which can be solved using a numerical method called monotone finite difference schemes: Monge-Ampere, Convex Envelope, Infinity Laplace, and Mean Curvature.

These methods have been implemented for registration of Brain Images. For Surface Registration, the Infinity Laplace equation is used to match surfaces using geodesic lengths [Sapiro]. For Volume Registration, the Monge-Ampere equation is used to minimize distortion of volumes [Tannenbaum-Haker-Haber]. Convergent numerical schemes are important in these applications: bad discretizations lead to artificial singularities in the mappings.

Focussing in on the Monge-Ampere equation, I'll show how naive schemes can work well for smooth solutions, but break down in the singular case. This makes having a convergent scheme even more important. I'll present a convergent solver which which is fast: comparable to solving the Laplace equation a few times.