
p-adic groups, Automorphic forms, and Geometry
Groupes p-adiques, formes automorphes et géométrie
(Org: **Clifton Cunningham** (Calgary) and/et **Julia Gordon** (UBC))

HESAMEDDIN ABBASPOUR, UBC

Indefinite Kac-Moody groups

While the affine Kac-Moody groups have been the focus of various researchers, the indefinite Kac-Moody groups have been mostly unexplored. I will try to give an overview of their construction and, time permitting, discuss my own research on the arithmetic theory of these groups.

PRAMOD ACHAR, Louisiana State University

Derived Satake equivalence and geometric restriction to a Levi subgroup

Let G be a complex reductive algebraic group. Let \mathcal{N} denote the nilpotent cone in its Lie algebra, and let Gr denote the affine Grassmannian of its Langlands dual group. The celebrated geometric Satake equivalence is an equivalence of tensor categories between spherical perverse sheaves on Gr and representations of G . Following methods of Arkhipov–Bezrukavnikov–Ginzburg, this can be extended to an equivalence of triangulated categories between the spherical derived category of perverse sheaves on Gr and the perfect derived category of coherent sheaves on \mathcal{N} . It is natural to ask, “Is this equivalence compatible with restriction to a Levi subgroup?” There are surprising subtleties involved in even making this question precise, essentially because the spherical derived category on Gr is the “wrong” category from the viewpoint of the Weil conjectures. I will explain these subtleties and how one may overcome them, leading to a positive answer to the question above. This is joint work with S. Riche.

MOSHE ADRIAN, University of Utah

Local Langlands correspondence - from real to p-adic groups

Given a connected reductive group \mathbf{G} over the reals \mathbb{R} , any Langlands parameter for $G(\mathbb{R})$ has image inside the normalizer of a maximal torus. Better, it has image inside the “E-group” of a maximal torus. The E-group of a torus is a slight generalization of the L-group of torus (it is not necessarily a semi-direct product), and is key to constructing the local Langlands correspondence for real groups. As Langlands parameters into the L-group of a torus correspond to characters of tori, Langlands parameters into the E-group of a torus correspond to genuine characters of two-fold covers of tori. Recently, Benedict Gross has developed an analogue of the E-group of a torus, for p-adic groups, called “groups of type L”. Langlands parameters into groups of type L (which generalize the Langlands parameters of DeBacker/Reeder) give something close to genuine characters of covers of tori, and in many cases they give genuine characters of two-fold covers of tori. We discuss this development and apply it to give a construction of the tame local Langlands correspondence for $PGSp(4, F)$ and $PGL(\ell, F)$, where ℓ is prime. We will then discuss how one might construct a local Langlands correspondence for more general groups using this theory. This work is joint with Joshua Lansky.

AMIR AKBARY, University of Lethbridge

Reductions of points on elliptic curves

Let E be an elliptic curve defined over \mathbb{Q} . Let Γ be a subgroup of rank r of the group of rational points $E(\mathbb{Q})$ of E . For any prime p of good reduction, let $\bar{\Gamma}$ be the reduction of Γ modulo p . Under certain standard assumptions, we prove that for almost all primes p (i.e. for a set of primes of density one), we have $|\bar{\Gamma}| \geq p/f(p)$, where $f(x)$ is any function such that $f(x) \rightarrow \infty$, at an arbitrary slow speed, as $x \rightarrow \infty$. This provides additional evidence in support of a conjecture of Lang and Trotter from 1977. This is a joint work with Dragos Ghioca (UBC) and Kumar Murty (Toronto).

BILL CASSELMAN, UBC

An explicit formula for the canonical pairing

The asymptotic behaviour of matrix coefficients determines a canonical pairing between two Jacquet modules. I'll present here an explicit version for representations induced from parabolic subgroups in terms of the pairing for the inducing representations. This generalizes Macdonald's formula.

CLIFTON CUNNINGHAM, University of Calgary

*Geometric construction of characters of \mathbb{Z}_p^**

Local class field theory (in a very simple case) tells us how to apprehend characters of \mathbb{Z}_p^* as characters of the inertia group for \mathbb{Q}_p . In this talk we explain how continuous characters of \mathbb{Z}_p^* may be identified with certain character sheaves on $\mathbb{G}_{m, \bar{\mathbb{Q}}_p}$, using Kummer-Artin-Schreier-Witt theory. We do this by exhibiting group schemes over purely ramified extensions of \mathbb{Z}_p that determine functors from Kummer local systems on $\mathbb{G}_{m, \bar{\mathbb{Q}}_p}$ to Artin-Schreier local systems on the special fibre of the group scheme, and then applying the sheaf-function dictionary; this is not Lubin-Tate. Under these functors, local systems of order d map to continuous characters of level $\log_p(d)$. The relation to class field theory for \mathbb{Q}_p will also be discussed.

Joint with Masoud Kamgarpour and Aaron Christie.

DRAGOS GHIOCA, University of British Columbia

p -adic analysis in algebraic dynamics

Using methods from p -adic analysis combined with arguments from nonarchimedean dynamics and from arithmetic geometry, we prove a gap principle in algebraic dynamics. Our result may be interpreted as an equivalent in dynamics of the classical Mumford's gap from Mordell's Conjecture.

EYAL GOREN, McGill University

Canonical subgroups for Shimura varieties

The canonical subgroup plays a key role in the study of overconvergent p -adic modular forms, and has been studied by many. I shall recall the problem and discuss a general (new) strategy for its study, which has already been proved optimal in the cases of Shimura curves and Hilbert modular varieties. Time permitting, I will discuss additional examples.

MATT GREENBERG, University of Calgary

p -adic interpolation and p -adic Jacquet-Langlands

In this talk, I will discuss applications of a p -adic Jacquet-Langlands correspondence to the interpolation of special values of L -functions of eigenforms varying in p -adic families. The resulting p -adic L -functions exhibit interesting exceptional zero phenomena.

MASOUD KAMGARPOUR, University of British Columbia

A family of Satake isomorphisms

Let F be a local field with ring of integers \mathcal{O} . Let G denote the general linear group and T the subgroup of diagonal matrices. In a remarkable 1973, Roger Howe defined a Satake-type isomorphism for every character of $T(\mathcal{O})$. If this character is trivial, we recover the usual Satake Isomorphism.

In this talk I will give an overview of Howe's construction and its applications to geometric representation theory.

GUILLERMO MANTILLA-SOLER, University of British Columbia
Mordell-Weil ranks in towers of modular Jacobians.

In this talk we describe a technique to bound the growth of Mordell-Weil ranks in towers of Jacobians of modular curves. In more detail, we will show our progress towards the following result. Let $p > 2$ be a prime, and let J_n be the Jacobian of the principal modular curve $X(p^{n+1})$. Let F be a number field such that $J_0[p] \subseteq F$. Then,

$$\text{rank} J_n(F) \leq 2[F : \mathbb{Q}] \dim J_n + o(\dim J_n)$$

for all n .

PAUL MEZO, Carleton University
Character identities in twisted endoscopy

The Local Langlands Correspondence motivates the definition of endoscopic groups attached to a reductive algebraic group. The representations of these endoscopic groups are conjecturally related to the representations of the initial reductive group through character identities. Such character identities have been proven in the case of real reductive groups. We outline the proof of twisted character identities, when an automorphism of the real reductive group is introduced into the theory.

DRAGAN MILICIC, University of Utah
Geometry and Unitarity

D-module techniques were very successful in helping to understand representation theory of real reductive groups. Until recently, they failed to help in explaining unitarity phenomena. Recent work by Vogan and his coworkers, and Schmid and Vilonen suggests that a geometric explanation is possible. As an illustration, we are going to discuss some basic examples. In particular, we are going to give a simple proof of the classic Segal-von Neumann theorem that connected noncompact simple Lie groups have no nontrivial finite-dimensional unitary representations.

HADI SALMASIAN, University of Ottawa
Fourier coefficients of Siegel Eisenstein series

I report on an ongoing project (joint with Eliot Brenner) on vanishing and nonvanishing results for Fourier coefficients of Siegel Eisenstein series for symplectic groups. The vanishing result follows from the theory of rank and singular representations, and one is naturally lead to the study of representations of the metaplectic group. The nonvanishing result is obtained by global methods.

LIOR SILBERMAN, UBC
A uniform spectral gap for congruence covers of a hyperbolic manifold

I will describe work with Dubi Kelmer on the first Laplace eigenvalue in towers of manifolds covered by real or complex hyperbolic n -space. All congruence quotients in a given dimension have a uniform spectral gap; we show how to deduce from this a uniform spectral gap for the family of congruence covers of a fixed arithmetic (non-congruence) manifold. A key ingredient is a lower bound on the dimensions of irreducible representations of groups defined over finite local rings.

JONATHAN SPARLING,
Nilpotents associated to Lie algebras with involution

We define certain nilpotent elements associated to a reductive symmetric space, and discuss an application of them to p -adic harmonic analysis.

LOREN SPICE, Texas Christian University

Harmonic analysis on p -adic SL_2

A series of papers by Sally and Shalika in the late '60's gave a stunningly detailed picture of harmonic analysis on p -adic SL_2 , but did not include the proofs for the character computations on which the enterprise was founded. 40 years later, "the Sally gang" gathered to use modern technology to create a streamlined version of the old proofs. In this talk, I will report on the tools we used, and interesting intermediate results along the way.

TASHO STATEV-KALETHA, Princeton University

L-packets and endoscopy for p -adic groups

We will discuss the construction of certain L-packets on extended pure inner forms of unramified p -adic groups and their endoscopic transfer.

ZHIWEI YUN, MIT

The Jacquet-Rallis fundamental lemma

I will explain the idea of the proof of the conjecture of Jacquet-Rallis in characteristic $p > 0$. The original problem can be reformulated in to a lattice counting problem in p -adic vector spaces, which can be further related to a global counting problem of vector bundles. Counting problems are shadows of sheaf-theoretic statements. I will explain the geometric ingredients in resolving the global counting problem.