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Infinitesimal non-crossing cumulants and free probability of type B

Free probabilistic considerations of type B have been around starting with a paper by Biane, Goodman and Nica from 2003. Earlier this year, Belinschi and Shlyakhtenko have connected free probability of type B to a framework called “infinitesimal free probability”. My talk will present a joint work with Maxime Fevrier (arXiv:0906.2017) where we pursue this interplay between “type B” and “infinitesimal”. Specifically, we introduce a concept of infinitesimal non-crossing cumulants (associated to an infinitesimal noncommutative probability space), which are obtained by taking a formal derivative in the formula for usual non-crossing cumulants. We prove that infinitesimal freeness of random variables is equivalent to a vanishing condition for mixed cumulants; this gives an infinitesimal counterpart for a theorem of Speicher from usual free probability. We observe how infinitesimal cumulants relate to non-crossing partitions of type B, and how they can be used to obtain infinitesimal analogues for some basic freeness results. We discuss some situations when freeness of unital subalgebras A_1, \dots, A_k in a noncommutative probability space (A, φ) can be upgraded to infinitesimal freeness (e.g., in the case when one has a derivation on A which leaves every A_i invariant).