Short Sums of Pretentious Multiplicative Functions

The literature is rich with asymptotic formulae for the sum of multiplicative functions $f(n)$ for $n \leq x$. In contrast, little is known about multiplicative functions summed over intervals $x < n \leq x + y$. We find asymptotic formulae for short sums of complex-valued multiplicative functions that are sufficiently “close” to 1 on primes $p$, and uniformly bounded on the prime powers. Some functions that fall into this category are $\sigma(n)/n$ and $\phi(n)/n$, where $\sigma$ denotes the sum of divisors function and $\phi$ the Euler totient function.