The residue class distribution of $\Omega(n)$

The Liouville function is defined by $\lambda(n) := (-1)^{\Omega(n)}$ where $\Omega(n)$ is the number of prime divisors of $n$ counting multiplicity. Let $z_m := e^{2\pi i/m}$ be a primitive $m$-th root of unity. As a generalization of Liouville’s function, we study the functions $\lambda_{m,k}(n) := z_m^{k\Omega(n)}$. Using properties of these functions, we give a weak equidistribution result for $\Omega(n)$ among residue classes. More formally, we show that for any positive integer $m$, there exists an $A > 0$ such that for all $j = 0, 1, \ldots, m - 1$, we have

$$\#\{n \leq x : \Omega(n) \equiv j \pmod{m}\} = \frac{x}{m} + O\left(\frac{x}{\log^A x}\right).$$

Best possible error terms are also discussed. In particular, we show that for $m > 2$ the error term is not $o(x^\gamma)$ for any $\gamma < 1$. Joint work with Sander Dahmen.