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Orthogonal and Biorthogonal Polynomials and integrable models of weakly dispersive water waves

One of the most studied in recent years nonlinear PDEs is the Camassa–Holm equation (CH) and its descendant, Degasperis–Procesi equation (DP), as well as a few other equations discovered within last year. One feature of these equations that stands out is that they possess interesting distributional (weak) solutions, non-smooth in a classical sense. Among them, the non-smooth solitons (peakons), first discovered by Camassa and Holm, exhibit remarkable mathematical features, which can be traced back to classical problems in the theory of orthogonal polynomials, essentially going back to T. Stieltjes. This connection becomes highly nontrivial for the case of the DP equation, for which the underlying polynomials belong to a new family of biorthogonal polynomials associated to the Cauchy kernel $1/(x+y)$. Fundamental properties of these biorthogonal polynomials were studied by M. Bertola, M. Gekthman and J. S. These polynomials can be used to construct explicit peakon solutions for the cubic version of the CH equation, the VN equation, discovered by V. Novikov. The last part of the talk gives an alternative construction of peakons for the VN equation given in the joint paper by A. Hone, H. Lundmark and J. S. (Dynamics of PDE, Vol.6, No.3, 253–289, 2009).