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Imprecise probabilities and exponential families

An imprecise probability law is a generalization of probability in which events may have upper and lower probabilities that are not equal. Such a construction may be a more realistic representation of prior ignorance than the (precise) prior distribution of Bayesian inference. Bayes' rule can be applied to imprecise probabilities to give posterior probabilities which are (hopefully) less imprecise than the priors. The imprecise Dirichlet prior was proposed by Walley as a practical solution to this updating process. Walley's model can be viewed as a subfamily of the Dirichlet family conjugate to the multinomial distributions of data. Whereas conventional Bayesian updating corresponds to a translation of a point in the hyperparameter space, imprecise updating translates a set of hyperparameters. The multinomial-Dirichlet conjugate family was generalized to other exponential families by Quaeghebeur and de Cooman. An imprecise probability model can be abstracted to a subset of an affine space of probability measures which is subject to random translations from observed data. Quantities of interest are typically expectations of parametric functions. In a useful imprecise model such expectations vary widely over the prior set of hyperparameters but exhibit a narrow range of values once this set is shifted by data.