
GAIL IVANOFF, University of Ottawa, 585 King Edward, Ottawa, ON K1N 6N5
Asymptotics for Spatial Causal ARMA Processes

The asymptotic behaviour of the empirical distribution $F_n(x) = \frac{1}{n} \sum_{i=1}^n I(X_i \leq x)$ of a stationary stochastic process (X_1, X_2, \dots) depends on whether the process is short- or long-range dependent. Generally, a process is defined to be short-range dependent if and only if the covariances $\rho_j = \text{Cov}(X_0, X_j)$ are summable.

A similar question can be asked about the empirical distribution of a spatial stationary stochastic process $(X_{ij}; i, j \geq 1)$. However, the spatial case is made more complex because the elegant martingale methods used in one dimension cannot generally be applied in higher dimensions. An exception is the spatial causal ARMA (autoregressive moving average) model, where $X_{ij} = \sum_{h=0}^{\infty} \sum_{k=0}^{\infty} a_{hk} \xi_{i-h, j-k}$; $\{\xi_{uv} : u, v \in \mathbf{Z}\}$ are i.i.d. random variables with mean 0 and variance 1 and $\{a_{ij}\}$ is an array of constants. We will review known results for the empirical distributions of one- and two-dimensional ARMA processes in both the short- and long-range dependent cases, and introduce a new central limit theorem for the empirical distribution of a spatial ARMA process in the short-range dependent case: i.e., when $\sum_{i \geq 0} \sum_{j \geq 0} |a_{ij}| < \infty$. We will show that due to the special structure of the spatial ARMA process, martingale techniques can still be applied.