Lie Groups and Automorphic Forms
Groupes de Lie et formes automorphiques

(Org: **Hadi Salmasian** (Ottawa) and/et **Wai Ling Yee** (Windsor))

JEFF ADAMS, University of Maryland

Involutions of Representations

The map $\pi \to \pi^*$ (contragredient) is an involution on the space of representations. If G is a reductive group over a local field, how do you compute this map in terms of L-homomorphisms? The same question can be asked for the map taking π to its "Hermitian dual"; the fixed points of this involution are the representations admitting an invariant Hermitian form. I'll discuss some answers to these questions.

This is joint work with David Vogan.

MOSHE ADRIAN, University of Maryland, College Park, Maryland, United States

Revisiting the Local Langlands Correspondence for $\mathrm{GL}(n, F)$, n a prime

The study of the Local Langlands Correspondence for $\mathrm{GL}(n, F)$, where F is a local field, was initiated by Jacquet–Langlands in 1970; many people have contributed since their work, and the proof has been completed by Henniart and Harris/Taylor. In the tame case, supercuspidal representations correspond to characters of elliptic tori satisfying certain regularity type conditions, but the local Langlands correspondence is unnatural because it involves a twist by some finite order character of the torus. Taking the cue from the theory of real groups, supercuspidal representations should instead be parameterized by characters of covers of tori. Stephen DeBacker has calculated the distribution characters of supercuspidal representations for $\mathrm{GL}(n, F)$, n a prime, and they are written in terms of functions on elliptic tori in $\mathrm{GL}(n, F)$.

Over the reals, Harish-Chandra parameterized discrete series representations of real groups by giving their distribution characters.

Those distribution characters are written in terms of functions on covers of real tori. I have succeeded in showing that if one writes down a natural analogue of Harish-Chandra's distribution character for p-adic groups, it is the character of a unique supercuspidal representation of $\mathrm{GL}(n, F)$, where n is prime, away from where the local character expansion is needed.

These results pave the way for a new statement of the local Langlands correspondence for $\mathrm{GL}(n, F)$, where n is prime. In particular, there is no need to introduce any character twists that occur in the already existing description of the local Langlands correspondence for $\mathrm{GL}(n, F)$.

DAN BARBASCH, Dept. of Mathematics, Cornell University, Ithaca, NY 14853

Dirac cohomology and unipotent representations

Dirac cohomology is an invariant of admissible representations generalizing $(\mathfrak{g}, K)$ cohomology, first introduced and studied by Huang and Pandzic. In this talk I will present results joint with P. Pandzic on computing this invariant for unitary representations for complex and real groups.

BIRNE BINEGAR, Oklahoma State University, Stillwater, OK 74078, USA

W-graphs and Primitive Ideals

Let G be a real form of a linear, reductive, complex algebraic group defined over $\mathbb{R}$. The W-graph of G is introduced first as a purely combinatorial object associated to the Kazhdan–Lusztig–Vogan polynomials for G. This graph is then reinterpreted from a purely representation theoretical point of view. Combining these two points of view we show how the set $\widehat{G}_{adm,\lambda}$ of irreducible admissible representations of regular integral infinitesimal character λ can be explicitly partitioned into equivalence classes sharing the same infinitesimal character.

ELIOT BRENNER, University of Minnesota, 206 Church St. SE, Minneapolis, MN 55455
Analytic Properties of Residual Eisenstein Series

We partially generalize the results of Kudla and Rallis on the poles of degenerate, Siegel-parabolic Eisenstein series to residual-data Eisenstein series. In particular, for a, b integers greater than 1, we show that poles of the Eisenstein series induced from the Speh representation $\Delta(\tau, b)$ on the Levi GL_{ab} of Sp_{2ab} are located in the "segment" of half integers X_b between a "right endpoint" and its negative, inclusive of endpoints. The right endpoint is $\pm b/2$, or $(b-1)/2$, depending on the analytic properties of the automorphic L-functions attached to τ. We study the automorphic forms $\Phi_i^{(b)}$ obtained as residues at the points $s_i^{(b)}$ (defined precisely in the paper) by calculating their cuspidal exponents in certain cases. In the case of the "endpoint" $s_0^{(b)}$ and 'first interior point' $s_1^{(b)}$ in the segment of singularity points, we are able to determine a set containing *all possible* cuspidal exponents of $\Phi_0^{(b)}$ and $\Phi_1^{(b)}$ precisely for all a and b. In these cases, we use the result of the calculation to deduce that the residual automorphic forms lie in $L^2\big(G(k)\backslash G(\mathbf{A})\big)$. In a more precise sense, our result establishes a relationship between, on the one hand, the actually occurring cuspidal exponents of $\Phi_i^{(b)}$, residues at interior points which lie to the right of the origin, and, on the other hand, the "analytic properties" of the original residual-data Eisenstein series at the origin. If time permits we will discuss further analytic properties such as wave-front sets of the residual automorphic forms, and applications of our calculations.

AARON CHRISTIE, Calgary

DAN CIUBOTARU, University Of Utah, Salt Lake City
On formal degrees for discrete series of classical affine Hecke algebras

The (expected) stability of L-packets of discrete series for p-adic groups implies that the formal degrees of the discrete series in the same L-packet have to be proportional. In Lusztig's category of representations with unipotent cuspidal support, this problem can be translated to one for affine Hecke algebras with unequal parameters. Following Reeder, Opdam, and Solleveld, the formal degree of a discrete series for affine Hecke algebras are known up to a rational constant (depending on the discrete series). Reeder conjectured a precise form for this constant, and verified this for the Hecke algebras arising for split exceptional groups. In joint work with Syu Kato, we compute the missing constants for the affine Hecke algebras of classical types with unequal parameters. The method of calculation is a consequence of a new algorithm for the W-structure of tempered modules for these Hecke algebras, based on Kato's exotic geometry.

This is joint work with Syu Kato.

CLIFTON CUNNINGHAM, University of Calgary
Cohomological compact restriction functors

Recently, Hadi Salmasian and I found cohomological analogues of the compact restriction functors taking depth-zero representations to representations of the reductive quotients of parahoric subgroups. Our functors take equivariant perverse sheaves on $G_{\bar{\mathbb{Q}}_p}$ to sheaf complexes on the reductive quotients of the special fibres of Bruhat–Tits schemes. In this talk I will present these functors and explain their significance in the study of admissible representations.

JULIA GORDON, University of British Columbia
Harish-Chandra characters and motivic integration

R. Cluckers and F. Loeser defined a class of objects, called constructible motivic exponential functions, which are defined in a field-independent way, by means of logic. Given a non-Archimedean local field K with a choice of a uniformizer and an additive character, they specialize to functions on K (or on varieties defined over K), when the residue characteristic of

K is sufficiently large. We expect that Harish-Chandra character of a supercuspidal representation is a specialization of a constructible motivic exponential function on the neighbourhood of the identity that is slightly larger than the neighbourhood on which Harish-Chandra's local character expansion holds, and we can prove it for a certain class of representations. I will discuss this result, and its potential applications to the question about local integrability of characters in positive characteristic.

Joint work with R. Cluckers, C. Cunningham, and L. Spice.

GERALD GOTTSBACHER, University of Toronto

On modular forms for orthogonal groups of rank 2

The construction of toroidal compactifications for non-compact Shimura varieties of orthogonal type allows to relate holomorphic differential forms on Kuga–Sato varieties, i.e., higher weight modular forms for $\mathrm{SL}(2)$, and modular forms for $O(n,2)$. I will describe the construction of the map providing this relation.

TATIANA HOWARD, University of Michigan, Ann Arbor, MI

Real Lie subalgebras of equal rank

It is known that if $\mathfrak{g}$ is a complex simple Lie algebra, then the maximal complex Lie subalgebras $\mathfrak{h}$ of rank equal to the rank of $\mathfrak{g}$ are obtained from the extended Dynkin diagram of $\mathfrak{g}$ by removing either one node with a prime label or two nodes with labels both equal to one (the label is the multiplicity of a given root in the highest root). Jeffrey Adams and I are working on an algorithm that will answer whether a given real form $\mathfrak{h}_0$ of $\mathfrak{h}$ is a subalgebra of a real form $\mathfrak{g}_0$ of $\mathfrak{g}$. This problem was solved by Berger for the special case when $\mathfrak{h}$ is obtained by removing a node with label equal to two. I will discuss our algorithm in the case when $\mathfrak{g}_0$ and $\mathfrak{h}_0$ are in the compact inner class.

BOGDAN ION, University of Pittsburgh, Pittsburgh, PA

On positive formulas for q-multiplicities

The only formula for q-multiplicities that manifests the non-negativity of their coefficients exists only in type A (Lascoux–Schutzenberger 1978). I will explain how positive formulas could be obtained in general. The statistic involved is the height of some special weights, called quasi-dominant weights which, in some sense, are highest weights for a degenerate enveloping algebra.

SARAH KITCHEN, University of Utah, Salt Lake City, UT 84112

Localization of degenerate principal series

For a Harish-Chandra pair (g,K), the Cousin resolution gives a geometric interpretation of the fact that the trivial representation can be written in the Grothendieck group of Harish-Chandra modules as an alternating sum of standard modules. I will give a more general geometric construction generalizing the Cousin resolution to partial flag varieties which allow for general homological methods to be used in the computation of composition series for some low rank examples of degenerate principal series.

FIONA MURNAGHAN, University of Toronto

Relatively supercuspidal representations

Let G be a connected reductive p-adic group. Let H be the fixed points of an involution of G. An irreducible smooth representation of G is said to be H-distinguished if there exists a nonzero H-invariant linear functional on the space of the representation. An irreducible smooth H-distinguished representation of G is said to be be H-relatively supercuspidal if all of the relative matrix coefficients of the representation have compact support modulo HZ, where Z is the centre of G. It is known that H-distinguished supercuspidal representations of G are always H-relatively supercuspidal. In general, there exist H-relatively supercuspidal representations of G that are not supercuspidal.

We will discuss some examples of H-relatively supercuspidal representations, and we will describe work in progress concerning construction of H-relatively supercuspidal representations.

MONICA NEVINS, University of Ottawa, Ottawa, Canada
Patterns in branching rules for representations of $\mathrm{SL}(2,k)$, k a p-adic field

Let k be a p-adic field of residual characteristic different from 2, with integer ring $\mathcal{R}$. The restriction of an irreducible representation of $\mathrm{SL}(2,k)$ to its maximal compact subgroup $\mathrm{SL}(2,\mathcal{R})$ is multiplicity-free but far from irreducible. We compare the irreducibles arising from restrictions of principal series and supercuspidal representations of $\mathrm{SL}(2,k)$ (due to previous work of the author) with known classifications of the representations of $\mathrm{SL}(2,\mathcal{R})$ (due to Shalika) to generate a picture of the information gleaned through such branching rules.

ANNEGRET PAUL, Western Michigan University, Kalamazoo, MI 49008
Unitary Principal Series of Split Real Groups

We present some recent progress in determining the unitary (minimal) principal series representations of split (odd) orthogonal groups, and the genuine unitary principal series of the metaplectic group. In particular, we assert that there is a natural injection of these two sets of parameters into the (known, due to work of Barbasch) set of data parameterizing the spherical unitary dual of certain other split real groups. We present some evidence to suggest that these injections are, in fact, bijections.

This talk is based on joint work in progress with Alessandra Pantano and Susana Salamanca-Riba.

SIDDHARTHA SAHI, Rutgers University
Littlewood–Richardson coefficients for Macdonald polynomials

Macdonald polynomials are a generalization of spherical functions of real and p-adic groups, and they are closely related to the representation theory of the double affine Hecke algebra. The type A Macdonald polynomials are a common generalization of Schur polynomials, Jack polynomials and Hall–Littlewood polynomials, which correspond to the general linear group over various fields. In this talk we describe some new results for these polynomials, including a recursive formula for the Littlewood–Richardson coefficients for the expansion of the product of two Macdonald polynomials.

YIANNIS SAKELLARIDIS, University of Toronto, 40 St. George Street, Toronto, ON M5S 2E4
On the Plancherel formula for spherical varieties over p-adic fields

Spherical varieties form a wide and interesting class of almost homogeneous spaces which includes symmetric ones. They have relevance to the Langlands program, and in fact we now have conjectures relating their spectrum to Arthur parameters of a distinguished type. I will describe several steps towards developing a Plancherel formula for spherical varieties of split groups over p-adic fields, which are in agreement with the conjectures. In many cases we obtain a Plancherel formula up to discrete spectra.

This is joint work with Akshay Venkatesh, and uses in a crucial way an unpublished argument of Joseph Bernstein.

SUSANA SALAMANCA, New Mexico State University, Las Cruces, NM 88003
On genuine omega regular unitary representations of $\mathrm{Mp}(2n)$

We discuss a class of genuine unitary representations of the Metaplectic group. We will present some results of the authors on the minimal principal series of $\mathrm{Mp}(2n)$ and then use them to parametrize the omega regular genuine unitary representations of this group. These are representations with infinitesimal character at least as regular as the metaplectic representation. The classification uses a generalized definition of $A_q(\lambda)$ representations. The proofs use a combination of arguments originally developed by Barbasch and Vogan and detailed in other publications by Pantano, Paul and Salamanca-Riba.

This work is joint work with A. Pantano and A. Paul.